Rigidity guided localisation for mobile robotic sensor networks

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Abstract: This paper introduces a rigidity-guided localisation approach for mobile robotic sensor networks. The localisation uses a distance graph composed of both the robot-to-robot ranging data and the motion trajectories from robot odometry. The motion of a robot depends on the result of the rigidity test of its local distance graph: if the graph is not uniquely localisable, the robot moves around in its neighbourhood to collect at least two extra ranging data with each of its neighbours in order to make the extended graph uniquely localisable. Locally unique maps are then merged into a globally consistent map.

Keywords: localisation; graph-rigidity networks; pebble-game; multi-dimensional scaling; ubiquitous computing; mobile sensor.

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1 Introduction

Recent advancements in wireless communication and Micro-Electro-Mechanical Systems (MEMS) have made possible the deployment of wireless sensor networks for many real world applications, such as environmental monitoring, search and rescue, military surveillance, and intelligent transportation, etc. (Akyildiz et al., 2002; Mainwaring et al., 2002; Simic and Sastry, 2003). The ability of a sensor node to determine its geographical location is of fundamental importance to sensor networks. Most of the existing localisation algorithms are developed for stationary sensor networks where the sensor nodes do not move once they are deployed. In recent years there has been growing interest in mobile sensor networks (Cortes et al., 2004) where all or a subset of the sensor nodes have motion capability endowed by robotic platforms. Mobile sensor networks have more flexibility, adaptivity and intelligence compared to stationary sensor networks. Mobile sensors can dynamically reposition themselves to satisfy certain requirements on monitoring coverage, network connectivity, or fault tolerance. In this paper, we focus on the positioning of mobile sensors.

Sensor networks are usually sparse either due to environment constraints (such as missing range measurements due to obstruction), or due to the limited number of sensor nodes. For sparse sensor networks, many of the existing localisation algorithms that depend on the range between nodes fail to work properly due to the lack of distance or connectivity data to uniquely calculate the geo-locations. Node mobility can be used to assist localisation. *Mobility-assisted localisation* (Priyantha et al., 2005) is to use one or more mobile sensors to add extra distance constraints to a sparse network by moving the mobile sensors in the area of deployment and recording distances to neighbours at these intermediate locations. Pathirana et al. (2005) developed a method based on Robust Extended Kalman Filter for a mobile node in a disconnected sensor network to estimate locations of sensor nodes it passes. For this purpose, one may also use more than one mobile nodes to add extra range measurements. For example, Virtual Ruler (Wang et al., 2006) uses two nodes attached to a mobile vehicle to achieve better localisation results in an indoor environment.

We assume a two dimensional mobile network consists of N robotic sensors. Among them only a small portion, M ($M \ll N$, for example, $M = 5\%N$) nodes, know their own locations. These nodes are called beacons, which may be equipped with GPS or other localisation techniques. With sensors (such as ultrasonic sensors), each node can measure the distance to its neighbours through the ranging techniques such as Time of Arrival (TOA) (Zhao and Guibas, 2004). The problem of localisation is as follows: *Given a distance graph* $G = \langle S_u, S_b, D \rangle$ *where* $S_u = \{x_1, x_2, \ldots, x_{N-M}\}$ *is a set of unknown nodes in an two-dimensional space*, $S_b = \{x_{N-M+1}, x_{N-M+2}, \ldots, x_N\}$ *is a set of* M *beacon nodes,* $D = [d_{ij}]$ *is the distance matrix. Find the* $N - M$ *unknown locations* S_u *such that* $|x_i - x_j| = d_{ij}$.

Wu et al. (2006a, 2006b, 2007), proposed a mobility-assisted localisation algorithm based on Multidimensional Scaling (MDS) (Ji and Zha, 2004) for mobile networks, in which each node can move around and measure the distances to its neighbours and the relative distances between successive positions along its trajectory. In this way, new constraints can be added by inserting virtual nodes, therefore, increasing the accuracy of the localisation. The results have shown significant improvement on sparse networks compared with the results when nodes are stationary. However, there are still two issues in that approach. Firstly, the number of virtual nodes to be added is empirically determined by the moving patterns. Secondly, the proposed approach did not take into consideration the realisability of the local patches. If the local patch of a node is already uniquely realisable, the existing constraints are already enough for uniquely determining the positions of the nodes in this patch; therefore no movement is needed. Unnecessary movement by the nodes in a network increases not only the time and energy cost of the localisation process, but also the error in the localisation due to the inherent cumulative noise in odometry tracking. In this paper, we propose a rigidity-guided localisation approach to address the above issues. The criteria for the movement of nodes and the addition of virtual nodes during the movement will be discussed. Note that although we use an MDS-based method for localisation after adding virtual nodes and extra distance measurements to a network, other localisation methods (such as those based on SDP (Biswas and Ye, 2004)) can be applied to the resulting network as well. It should be noted that the proposed method is a distributed method based on patch merging. It does not have to have anchor nodes at all if we are only interested in the relative positions of the nodes in the network.

This paper is organised as follows. Section 2 discusses the related work in sensor network localisation. Section 3 gives a brief introduction to the MDS-based localisation. Section 4 gives a brief introduction to the rigidity theory and rigidity checking. Section 5 presents the proposed approach of rigidity-guided localisation.

2 Related work

In recent years, various local positioning systems (Kolodziej and Hjelm, 2006; Pfeifer and Elias, 2003) and localisation methods have been developed for ad hoc wireless sensor networks. Most of the node localisation algorithms are based on range measurements, through either TOA (Zhao and Guibas, 2004), Time Difference of Arrival (TDOA) (Savarese et al., 2001), Received Signal Strength (RSS) (Bulusu et al., 2000; Nguyen et al., 2005) or multi-modal sensing (Girod and Estrin, 2001).

Niculescu and Nath (2001) proposed the Ad-hoc Positioning System (APS), which is based on the triangulation technique and is a distributed protocol that requires reasonable memory and message overhead. APS assumes that there are at least three anchor nodes, whose positions are known, in a sensor network. Every other sensor tries to find out its distance to the anchor nodes. When the distance information to three or more anchor nodes is obtained, the sensor node can compute its own position using triangulation. DV-hop and DV-distance are the two common methods for finding the distances between a node and the anchor nodes. Both DV-hop and DV-distance measure distance in a hop-by-hop manner. However, both DV-hop and DV-distance are only approximation of the physical distances between nodes, which inevitably leads to error in the localisation. To reduce the error, Lim and Hou (2005) proposed to use proximity-matrix to get a better estimate of the physical distance from the path distance. The Proximity-Matrix (PDM) is derived from the path distances between anchors and the physical distances between the anchors, and captures the topological information of the network. However, for anisotropic networks, Cheng et al. (2006) showed that Lim's method can not always return better localisation than APS and proposed a hybrid method in which each node independently chooses the better method to use based on analysing the incoming information. Doherty et al. (2001) proposed a localisation method based on convex optimisation in which the position estimation is based exclusively on connectivity-induced constraints and a rectangular bound around the possible position for each node in the network can be obtained. Statistical models of TOA, AOA, and RSS, and their use to generate localisation performance bounds were discussed in Patwari et al. (2005).

Centralised algorithms have also been developed for network localisation. Shang and Ruml (2004) proposed a centralised 3-step method, namely the MDS-MAP. This algorithm first uses an all-pair shortest-path algorithm to get a rough estimation of the distance between all possible pairs of nodes. Then, the algorithm proceeds to apply the MDS to derive node locations fitting the initially estimated distances. Lastly, with the absolute positions of three or more anchors, the relative map can be transformed into the absolute coordinates of all the sensor nodes. Biswas and Ye (2004) proposed a Semi Definite Programming (SDP) based method for the localisation in wireless sensor networks. The optimisation problem is set up so as to minimise the error in sensor positions for fitting the distance measures. The basic idea behind the technique is to convert the non-convex distance constraints into convex constraints. Recently, there are several works in distributed localisation of rigid networks, such as Patchwork (RevKoren et al., 2005), ARAP (Zhang et al., 2009), and locally rigid embedding (Amit, 2008). In these work, it is assumed that the local patches around each individual node is rigid.

There also exists work on solving localisation, tracking and mapping problems for mobile robots in robotics, which heavily relies on sophisticated sensors such as sonar, laser range finder, or camera on-board the mobile platforms (Spletzer, 2003). However, most of these mobile sensors have very stringent constraints on the cost and complexity. Tilak et al. (2005) developed dynamic localisation protocols for mobile sensor networks. However, their main interest is on how often the localisation should be carried out in a mobile sensor network and not on the localisation method itself. Recently, Hu and Evans (2004) proposed Sequential Monte Carlo (SMC) localisation method to solve the localisation problem and they found that the mobility of the sensors can be exploited to improve the accuracy of localisation. Using a similar approach, Simultaneous Localisation, Calibration and Tracking (SLAT) of a mobile node within a set of static sensor nodes has been developed (Taylor et al., 2006), where both the mobile node and the set of static sensor nodes are localised using range measurements. Recently, a lot of work has been done using sensor networks for tracking mobile targets (Vercauteren and Wang, 2005; Vercauteren et al., 2005; Liu et al., 2007; Wang and Wang, 2007; Lin et al., 2006; Zou and Chakrabarty, 2007; Tsai et al., 2007; Tseng et al., 2004; Zhang et al., 2005).

The accurate localisation of a network is closely related to the rigidity of the network given the existing constraints. Jacobs and Hendrickson (1997) discussed the rigidity of two-dimensional graphs and proposed a polynomial algorithm (Pebble games) to test whether a graph is rigid. Eren et al. (2004) discussed the conditions for a network to be uniquely localisable and proposed a method for constructing uniquely localisable networks using a mobile robot. Goldenberg et al. (2005) discussed the identification of the rigid components in partially localisable networks.

3 MDS-based localisation

Classical Multidimensional Scaling (Borg and Groenen, 1997; Ji and Zha, 2004; Costa et al., 2006) is a popular statistical tool used for data analysis, where the dimensions of the data are reduced suitably to retain only the important dimensions which would be good enough to reproduce most of the information contained in the data. Though MDS is primarily used in data mining, it does find a lot of applications in other fields as well and most notably, in localisation problems associated with wireless subjects. Based on the measurement level of the proximity variable on which the objects are differentiated, MDS is further classified into Metric MDS and non-Metric MDS. The localisation problem discussed here comes under the category of metric MDS.

The power of this algorithm lies in its ability to depict the dissimilarities between the objects through a placement of points in a low-dimensional plane where the Euclidean distances between the points resemble the actual proximity between the objects as closely as possible. In the case of localisation problems, the dissimilarity measurement of n subjects is an $n \times n$ distance matrix. Given this matrix, MDS can then plot these points using the centroid as the origin. It requires at the most $n-1$ dimensions to represent the proximity perfectly, which is not advantageous most of the time. To get a perceivable output, there must be just 2 or 3 dimensions which are good enough to contain most of the information. Hence singular value decomposition is carried out on the distance matrix and only those dimensions that convey most of the information are preserved. In mathematical terms, these are the dimensions which are associated with the most significant eigenvalues. In summary, the MDS-based localisation algorithm is as follows (Shang and Ruml, 2004):

1. Initialise the distance matrix with distances between neighbouring node pairs. If there is no distance information, an '*infinite*' value is assigned as the distance.
2. Floyd's Algorithm (Atallah, 1998) is used to compute the shortest path distance between each pair of nodes using the connectivity information.
3. The symmetric distance matrix obtained in the above step is input to the Classical MDS (CMDS) algorithm. CMDS eliminates dimensions corresponding to non-significant eigenvalues, thereby constructing a relative map with 2 or 3 dimensions.
4. The relative map obtained is transformed into an absolute map, if provided with sufficient number of beacon nodes, (3 nodes for a 2-D and 4 nodes for a 3-D networks). First, a transformation function is created by mapping the relative coordinates of the beacons with their known absolute coordinates. The obtained transformation function is then applied to the rest of nodes. An optional refinement step (Shang and Ruml, 2004) involving least-squares minimisation can be included to adjust the relative maps so that the inter-distances of the nodes conform to the measured distances.

To cope with irregular network topology, an improved MDS-MAP algorithm (Shang and Ruml, 2004) is proposed. In this algorithm, each node constructs a local map which only includes its one or two-hop neighbours using CMDS. The local maps are then merged to get the global map of the whole network.

4 Network localisability and rigidity

In this section, we are going to introduce the theory of network localisability and rigidity, on which the proposed method is built on. A detailed description can be found in Jacobs and Hendrickson (1997), Eren et al. (2004), and Goldenberg et al. (2005).

Let $G = \{V, E\}$ denote a network of vertices $V = \{1, 2, \ldots, n\}$ and for any edge $(i, j) \in E$, the distance between V_i and V_j is precisely known. The network localisation problem is to determine the unique position of each node in the network given the positions of available beacons and the distance between each pair $(i, j) \in E$. If under the given constraints, there is only one position for each node, then the network is localisable. The network localisation problem is closely related to the Euclidean graph realisation problem, in which coordinates are assigned to vertices of a weighted graph such that the distance between coordinates assigned to nodes joined by an edge is equal to the weight of the edge.

For a two dimensional graph with N vertices, the positions of its vertices have $2N$ degrees of freedom, of which three are the rigid body motions. Therefore the graph is rigid if there are $2N-3$ constraints. A graph is rigid if the location of nodes in it can not be changed continuously under the existing edge constraints.

If each edge adds an independent constraint, then $2N-3$ edges should be required to eliminate all nonrigid motions of the graph. In other words, for a network with N nodes, there are at most $2N-3$ independent edges. Therefore, if any induced subgraph with N vertices has more than $2N-3$ edges then these edges cannot be independent, which leads to the following Laman theorem (Laman, 1970):

Theorem 1: *The edges of a graph $G = \{V, E\}$ are independent in two dimensions if and only if no subgraph $G' = \{V', E'\}$ has more than $2N' - 3$ edges, where N' is the number of nodes in G'.*

Corollary 1: *A graph with $2N - 3$ edges is rigid in two dimensions if and only if no subgraph G' has more than $2N' - 3$ edges.*

Laman's theorem characterises rigidity. However, a direct implementation of it leads to a poor exponential algorithm. Jacobs and Hendrickson (1997) proposed an efficient approach to check for rigidity based on a pebble game. Jacobs' approach uses the following formulation of Laman algorithm:

Theorem 2 (Jacobs and Hendrickson, 1997)**:** *For a graph $G = \{V, E\}$ having M edges and N vertices, the following are equivalent.*

- *The edges of G are independent in two dimensions.*
- *For each edge (a, b) in G, the graph formed by adding three additional edges identical to (a, b) has no induced subgraph G' in which $M' > 2N'$ where M' and N' are the number of edges and nodes respectively in G'.*

The basic idea behind Jacobs' algorithm is to grow a maximal set S of independent edges one at a time. Initially, S is empty. A new edge is added to S if it is discovered to be independent of the edges existing in S. To check whether an edge e is independent of edges in S, each vertex is assigned two pebbles initially and a temporary set S' is created. S' contains all the edges in S plus four copies of e. If all edges in S' can be covered by the pebbles, then we know that e is independent of all edges in S and e is added into S. This process is repeated until no more edges can be added into S. Then S is a maximal set of independent edges. If S contains $2N - 3$ edges, then the graph is rigid. Figure 1 shows the initial pebble for a four-node network and the pebble covering for checking the whether edge AD is independent of all other edges in the network. The dashed edges are the duplicates of edge AD. Using pebble game to check independent edges.

Having $2N - 3$ independent edges ensures the rigidity of a graph. However, it does not guarantee the unique realisation of the graph. A discontinuous change to the positions of nodes may lead to another realisation which satisfies all the constraints of the network, as shown in Figure 2. A rigid graph subject to discontinuous transformation. The following theorem states the condition for a network to be uniquely realisable.

Theorem 3 (Jackson and Jordn, 2005)**:** *A graph G with $N \geq 4$ vertices is uniquely realisable in two dimensions if and only if it is redundantly rigid and tri-connected.*

Redundant rigidity means that after removing any single edge, the remaining graph is still rigid. A tri-connected graph is a connected graph such that deleting any two vertices (and incident edges) results in a graph that is still connected. When a network satisfies the condition in Theorem 3, it can be uniquely localised given at least three nonlinear beacons in two dimensional space.

Figure 1 The left figure shows a network with four nodes and each node has two pebbles. The right figure shows one possible pebble covering for the edges when checking edge AD, assuming the other edges are already in the independent edge set. The arrow points away from the node whose pebble is used to cover the edge. This pebble covering proves that edges AD is independent of edge AB, BC, CD, and BD

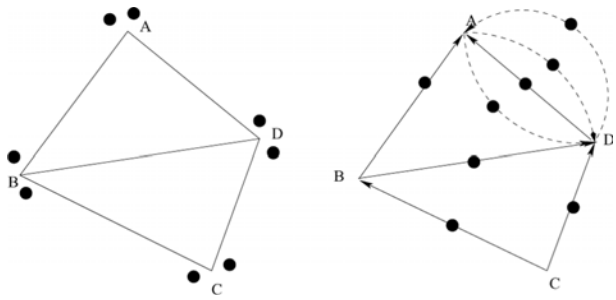
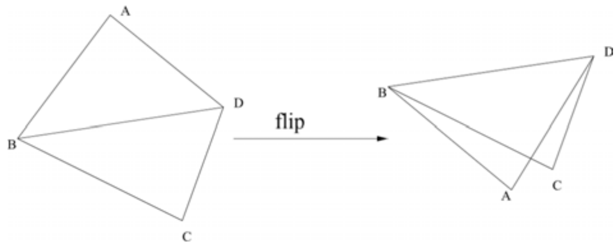


Figure 2 All the possible subgraphs do not exceed the limit number of edges shown in Corollary 1, but the graph is not uniquely realisable. The two realisations are not continuous in two dimension space in that the second one is obtained by a flipping of the first one



5 Rigidity guided localisation

In this section, we will present a new approach for localising a mobile network with the guidance from rigidity check. In this proposed approach, virtual nodes are added through the movement of mobile nodes. The newly added virtual nodes introduce additional constraints, which help turn the otherwise unlocalisable network into a uniquely localisable one. The details of the proposed approach is described in Section 5.1. Section 5.2 discusses the localisability of the network with added virtual nodes.

5.1 Localisation guided by rigidity

When the network starts a localisation process, each node will contact its neighbours to get all the connection information as well as the distances between them. With this information, each node can construct a local patch, which will be merged with the local patches of other nodes in order to get the final localisation of the whole network. The coordinates of nodes in a local patch is computed through Classical Multiple Dimensional Scaling (CMDS). When the local patch is uniquely realisable, more accurate localisation can be obtained by adding a refinement processing following CMDS. In the case that the local patch is not rigid, then large localisation error can happen due to insufficient number of constraints. For a mobile network, additional constraints can be obtained by moving the nodes in their neighbourhood and adding virtual nodes. However, not all the nodes have to move around. Only a node whose local patch is not uniquely realisable has to move.

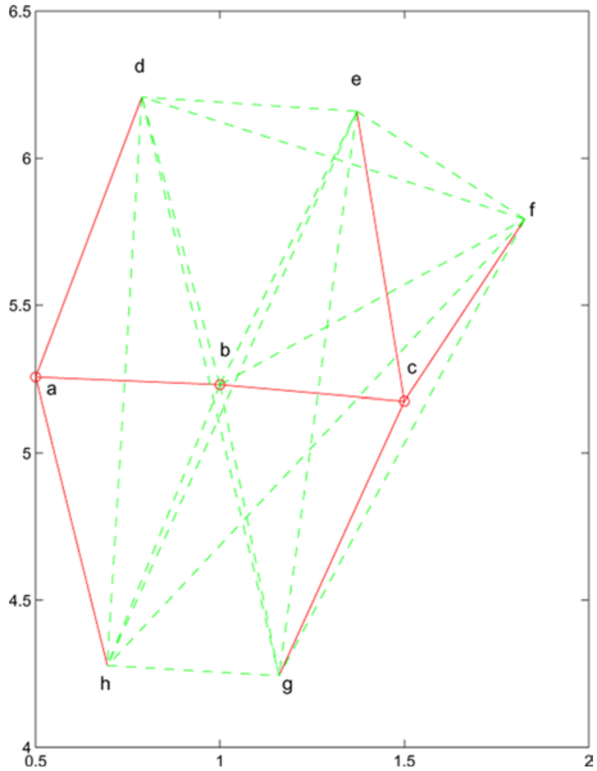
Since the localisation is based on the distance measurements between nodes, it would be appropriate to define the connectivity between nodes based on the range of reliable distance measurement, rather than the communication range. For the rest part of this paper, we refer to the maximum distance within which reliable distance measurement can be obtained as r. Nodes n_1 and n_2 are not one-hop neighbours if there are no distance measurement between them, i.e., $(n_1, n_2) \notin E$.

When the localisation process starts, each node collects the edge information from its local neighbourhood, in our case, the two-hop neighbours and check whether the local neighbourhood is already redundantly rigid and tri-connected. If yes, then no movement is needed. Otherwise, it should start moving in a circle, which is centered at its original location and of radius equal to r. During the movement, it periodically scans its neighbourhood and measures the distances to its neighbours. The moving node adds a virtual node when one of the following two conditions is met:

- it sees a neighbour for the first time
- a neighbour disappears from its view for the first time.

In the first case, a moving node should record the ID of this neighbour, the distances to it, and create an ID for the virtual node. This neighbour also records the ID of the virtual node and the distance to the virtual node. In the second case, the moving node uses the measured distance of the last scan as the distance between the virtual node and the neighbour that disappeared. If the moving node scans its neighbours frequently enough, the distance between its current position and the last position where it can see the node that just disappeared is very small. Therefore, it is safe to use the last distance measurement as the estimate of the distance between the moving node's current position (virtual node) and the node that disappeared. Figure 3 shows the virtual nodes added by the movement of the middle node in a 3-node network. The virtual nodes added by the movement of the middle node.

Figure 3 The original nodes (a, b, c) are denoted by small circles. d, e, f, g, h are the virtual nodes added during the movement of b. The edges obtained by range measurements are shown in solid lines. The edges from odometry are shown in dashed lines. The radius of the trajectory of b is 1. After adding the virtual nodes, the local patch around b, including both the virtual nodes and the original nodes is uniquely realisable (see online version for colours)



When the localisation process starts, each node checks the rigidity of its local patch and determines whether it needs to move. The nodes can move sequentially in a one-by-one order or they can move in parallel. In the later case, to avoid unnecessary complexity, only nodes between which the distance is more than two-hops can move simultaneously. This can be done in the following way. When a node finds that it needs to move, it sends out an ABOUT-TO-MOVE message with its ID to all of its two hop neighbours. When a node receives an ABOUT-TO-MOVE message, if it does not have to move, it can simply send an ACK message back to the sender. Otherwise, it compares its ID with the ID of the sender. If its ID is smaller, then it sends an ACK message back, otherwise, it sends a NACK message back. Once a node receives ACK messages from all of its two-hop neighbours, it can start to move. When it finishes movement, it sends out a MOVE-FINISHED messages to its two-hop neighbours.

After all nodes have finished a round trip (practically, the nodes cannot get back exactly to their original positions due to odometry and control error, the algorithm should work as long as the offset to its original position is not too big. In simulation, we use odometry error to take that into account), each node can build its local map using the distances between nodes including virtual nodes in its neighbourhood. The local map of a node n only contains the two-hop neighbours of n in the original network before the movement and the virtual nodes added during the movement of n. The local map is obtained by refining the map determined from CMDS (Wu et al., 2006b). The local maps can then be merged either sequentially or in parallel. There are various ways of merging local maps sequentially, such as randomly or according to certain order best for an application. The merging will stop when all the nodes excluding the virtual nodes are included in the global map. Note that virtual nodes will not be merged, they are only used for computing local maps. Given sufficient beacon nodes (3 or more for 2-D networks) the global map can be transformed to an absolute map based on the absolute positions of the beacons.

The proposed approach can be summarised in the following steps:

- Each node n collects its neighbours' information and checks the unique realisability of its local patch.
- If the local patch is not uniquely realisable, n starts to move in a circle about r and adds virtual nodes during the movement.
- Given the available distance measurements, n builds its local map using CMDS followed by a refinement process. It should be noted that methods other than CMDS can be used.
- The local maps are merged in a way similar to the one in Shang and Ruml (2004) to get a global map which includes all real nodes in the network.
- Given sufficient number of beacons, the global map is transformed into an absolute map.

It should be noted that this paper focuses on the distributed localisation of a network, in which the local patches are merged to get the global localisation. It is therefore necessary to check the localisability of the local patches to avoid unnecessary movement. The goal is not to find the minimum movement for localising the network but to get correct localisation. When a centralised approach is used, the whole network can be first checked to see whether it is already uniquely realisable or the uniquely realisable component of the network can be identified to reduce unnecessary movement. However, this paper does not focus on the centralised approach.

5.2 *Rigidity of a network after adding virtual nodes*

In this section, we would like to prove that the proposed approach of adding virtual nodes leads to unique realisations for local patches and a global map. It is very common that mobile robots have motion tracking sensors such as optical encoders for tracking the heading and the velocity. Therefore the relative positions between virtual nodes and the original position can be obtained. If we assume that all moving nodes have this capability, we have the following theorem:

Theorem 4: *For each node n with at least one neighbour, the local patch including n, the original two-hop neighbours, and the virtual nodes created during its movement has a unique realisation if nodes less than r from n are one-hop neighbours of n.*

Proof: Let n_1 be a one-hop neighbour of n. Let $P=\{p_i\}_{i=1}^{m}$ be the virtual nodes added during the movement of n. According to the way virtual nodes are added, m is not less than two. Since the node can track its positions during movement reliably, then the distance between every pair of virtual nodes is known and the distance between every virtual nodes to the original position of n is also known. Therefore the virtual nodes together with n forms a complete graph G_n, which has at least three nodes. Obviously, a complete graph always has a unique realisation up to rotation and translation transformation. For n_1, at least two virtual nodes p and q are connected to it: p is added when n sees n_1 for the first time, and q is added when n_1 disappears from the sight of n for the first time. Therefore, n_1 has at least three edges connecting it to G_n, therefore n_1's position is uniquely determined, with respect to G_n. Following this logic, all one hop neighbour of n is uniquely realisable after the movement of n.

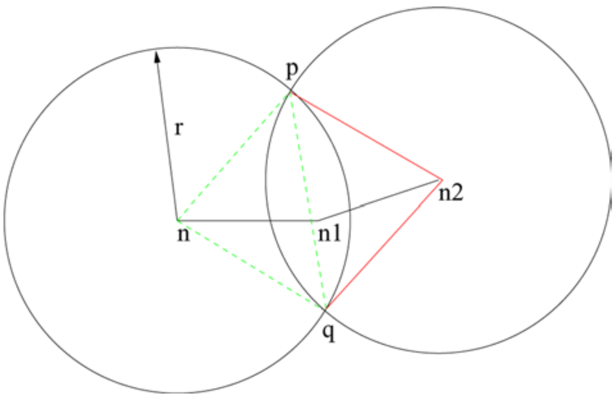
Let n_2 be a two-hop neighbour of n, as shown in Figure 4. It is connected through an one-hop neighbour of n. Without loss of generality, let the intermediate node be n_1. Since the radius of the circle is r, n_2 will have two edges connecting to G_n as well, and one edge connecting to n_1. Since the position of any one-hop neighbour is uniquely determined, then the position of n_2 as a two-hop neighbour is also uniquely determined. This proves that all nodes within two-hops of n in the local patch are uniquely realisable after adding virtual nodes and extra distance measurements. Proof of the rigidity of the network after adding virtual nodes. □

Based on the unique realisability of the local patches, it is straightforward to prove the following theorem:

Theorem 5: *The networks including all original nodes, edges, virtual nodes and the edges introduced by virtual nodes are uniquely realisable.*

Proof: As the previous theorem has stated, the local patch of any node with more than one neighbour is uniquely realisable. Since the network is connected, the map of the whole network can be constructed by merging the local patches until all nodes, excluding the virtual nodes, are included. For a network with more than three nodes, the local patches of two one-hop neighbours n_1 and n_2 will have at least three nodes in common. If the common nodes are not linear, then the two patches can be merged. The construction process can start from a local patch of n, and merge the local patch of n with the local patch of n's neighbour to get a bigger patch R if there are more than three nonlinear common nodes. Repeat the process until R contains all nodes excluding virtual nodes. Since during each merging, the transformation is unique, and each patch is uniquely realisable up to only translation and rotation, the final patch R is also uniquely realisable up to translation and rotation only. □

Figure 4 n, n_1, and n_2 are the original nodes and n is the moving node. p and q are two virtual nodes added for node n_2 by n. The dashed edges are introduced by the odometry of n and the solid lines represent the edges added by range measurement between n and n_2 during n's movement. n_2 now has three edges to the graph formed by the virtual nodes, n, and n_1. Therefore n_2 becomes localisable (see online version for colours)



Up to now, we assume that nodes move in a circular way. In real application such as indoor environments, this assumption may not hold. But this does not invalidate the proposed approach since the node can move in any way that is suitable as long as it sees every node in its two-hop neighbourhood at least two times.

As long as each node n in the two-hop neighbourhood of the mobile node is seen twice during the travel of the node, two virtual edges will be added, thus turning n to be part of the uniquely realisable local patch. If a neighbour is missed being seen, another traverse in the area may be conducted.

6 Experiments

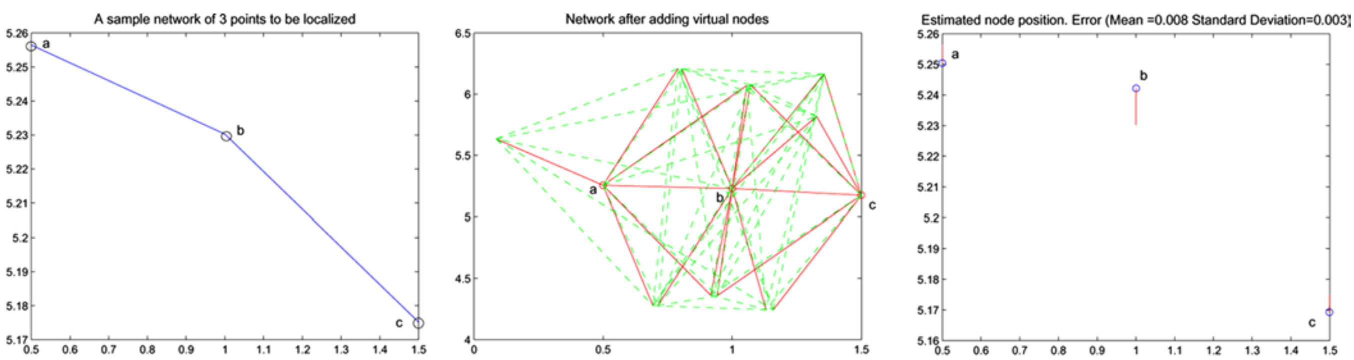
In this section, we evaluate the performance of the proposed localisation approach in various cases. We are interested in how the proposed approach can localise sparse networks. We tried to evaluate the proposed approach from the following aspects:

- whether it can turn sparse network into localisable networks
- what is the effect of noise in the localisation using this approach
- what is the relationship between number of moving nodes and the network average degree.

The network average degree is the average of the connectivity degree of all nodes in the networks.

In our experiments, we consider the two types of multiplicative errors that commonly exist in localisation. One is the ranging error and the other is the odometry error. We assume that the ranging error is linear to the actual distance between nodes and that the odometry error is cumulative. For simplicity purpose, we model the relative error of both ranging error and odometry error as a normal random variable having the distribution $N(0, \sigma^2)$. Let d be the distance between node i and j, then the error of the range measurement between i and j is defined as $\rho * d$, where ρ is a random number from distribution $N(0, \sigma^2)$. Similarly, let p_i be the real position of the previous virtual node added by a moving node and p_j be the real position of current virtual node, then the odometry error happened during the movement from p_i

Figure 5 This figure shows that with the help of added virtual nodes, the low degree networks can be accurately localised: (a) A three node network to be localised; (b) The network after adding virtual nodes. The solid edges are added by the range measure. The dashed edges are added by the odometry of the moving nodes and (c) The localisation result. The localisation error is indicated by a red line segment starting from the known position to the estimated position by the localisation (see online version for colours)



(a) (b) (c)

to p_j is $\rho * |p_i - p_j|$. Note that this is only a simple model. In real platforms, ranging error can be either larger or smaller than odometry error, and odometry error is more or less scaled with length or angle it travelled, not the distance between two points.

Figures 5 and 6 show the experiment results of localising a three-node network and a six-node network, respectively. The average degrees of the networks are no more than 2. The deviation of the errors added in the range measurement and the odometry is 5% and r is set to 1.0. In Figures 5(c) and 6(b), the localisation error is indicated by a line segment starting from the known position to the estimated position. We draw a circle around the estimated positions of nodes to make them easier to be observed. A similar representation is used in other figures in this paper. As we can see,

Figure 6 The localisation of a sparse six-node network. The mean localisation error is 1.8% of r and the deviation of the localisation error is 1.3% of r: (a) a network with six nodes and (b) estimated positions (see online version for colours)

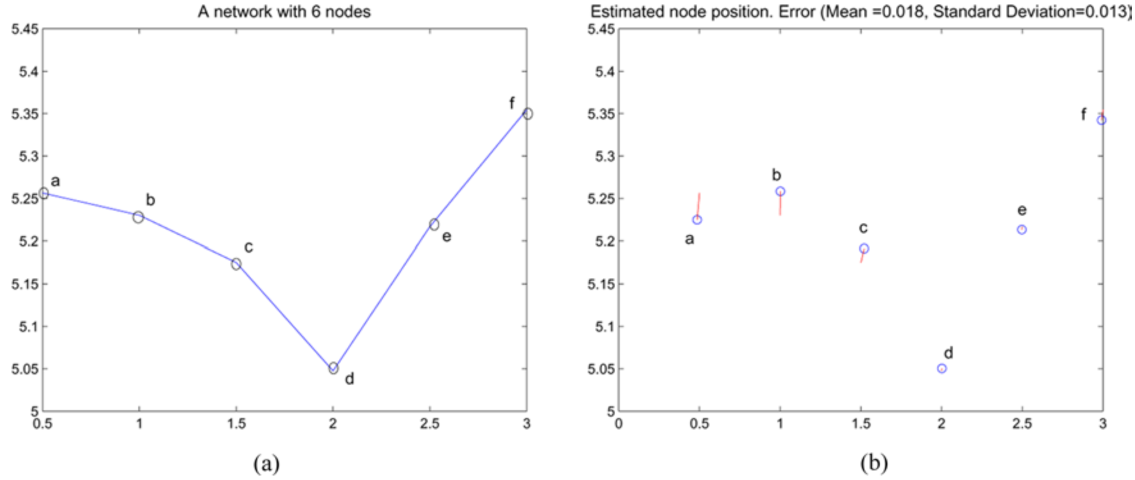
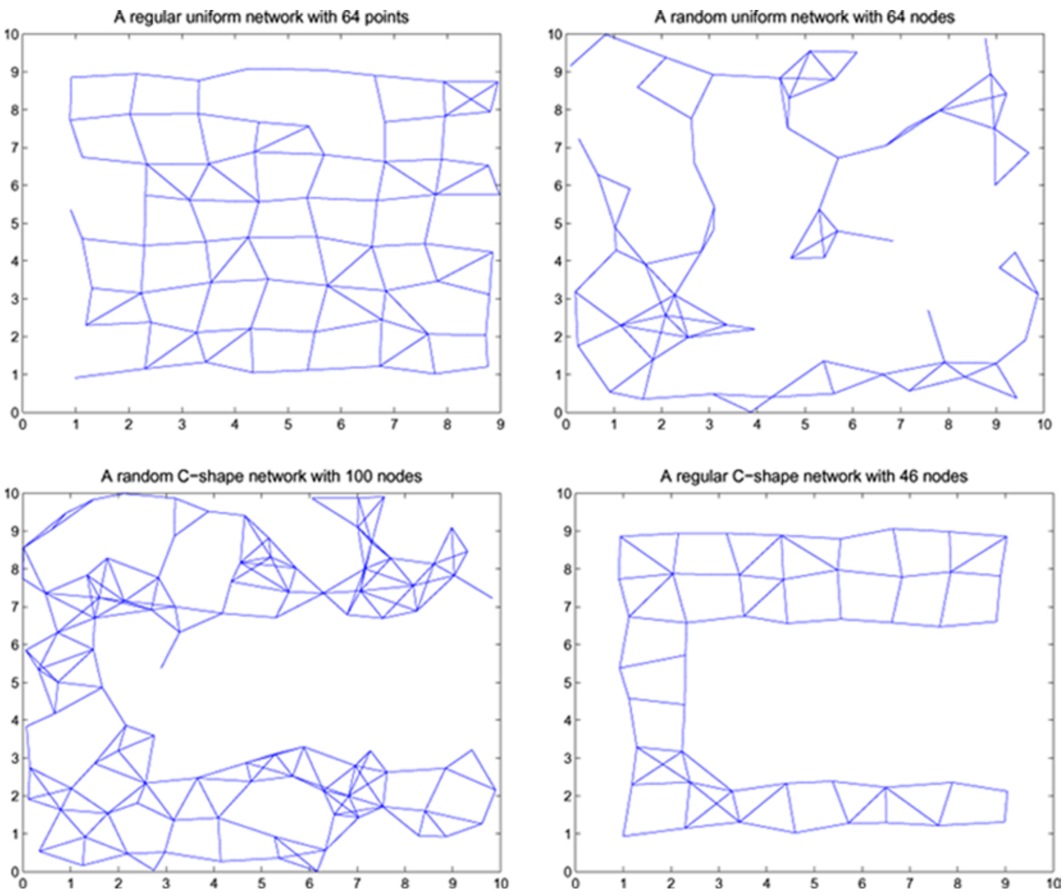


Figure 7 The four ground truth networks of different topologies for evaluating the performance of the proposal approach. From left to right, they are regular uniform, random uniform, random C-shape, and regular C-shape. The estimated positions of nodes will be compared with the known positions in the four networks (see online version for colours)



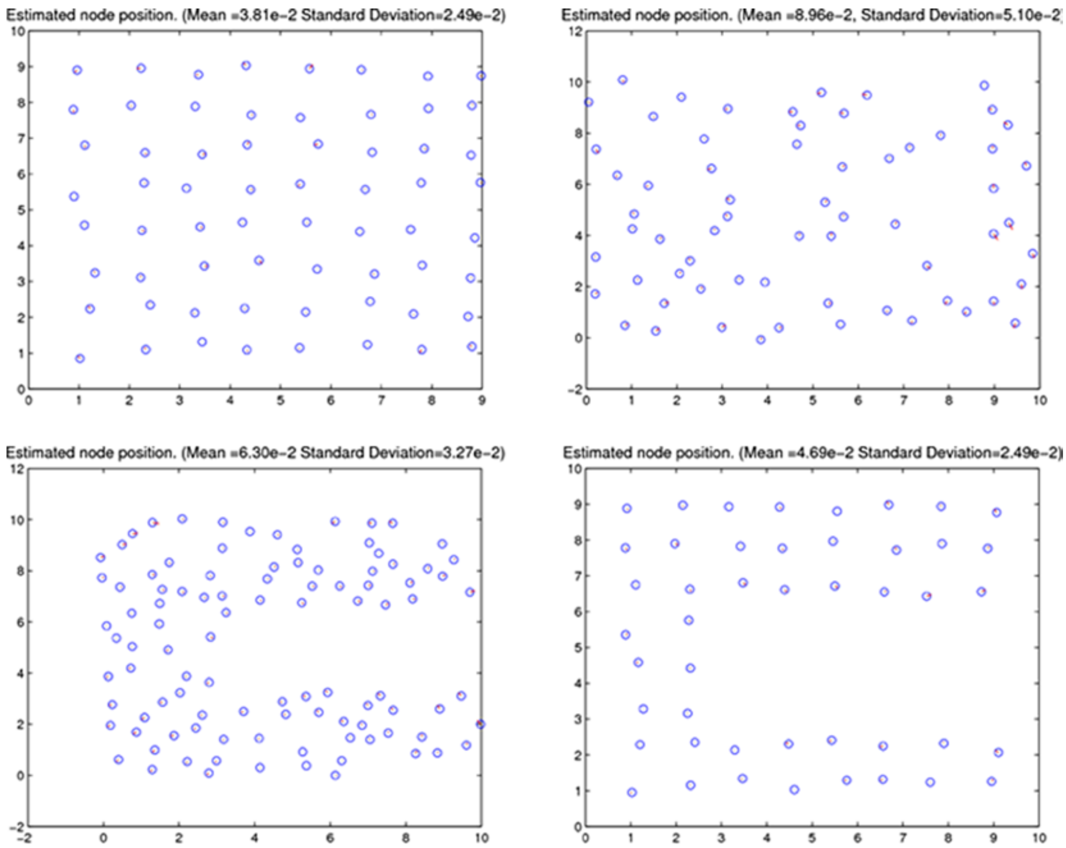
the added virtual nodes turn networks of such low degrees into localisable networks. The localisation error is less than 2% of r in both cases. The results from the two experiments are consistent with Theorem 4. If the network is connected, then according to Theorem 5, the whole network is localisable through adding virtual nodes by movement of nodes.

To further confirm the performance of the proposed approach, experiments have been carried out on four types of networks shown in Figure 7: regular uniform, random uniform, random C-shape, and uniform C-shape. The estimated positions of nodes will be compared with the known positions in the four networks. The four networks resemble networks of various topologies. In this experiment, r is chosen to be as small as possible but also big enough so that a connected graph can be easily generated. Smaller r leads to sparser network. In the regular uniform case, there are 64 nodes. r is 1.3 and the average degree is 3.71. Among the 64 nodes, 62 nodes or 96.8% of nodes have to move. In the random uniform case, there are also 64 nodes, r is 1.5 and the average degree 3.1875. In this case, 61 nodes have to move. In the regular C-shape case, there are 46 nodes, r is 1.5 and the average degree is 3.65. In this case, all nodes participated in the movement. In the random C-shape case, there are 100 nodes, r is 1.3 and the average degree is 4.44. Among the 100 nodes, 70 nodes have to move. Table 1 shows the results when no ranging error or odometry error is considered. The localisation error is extremely small, which conforms to Theorem 5. Figure 8 shows the localisation results when multiplicative ranging and odometry errors are added. The deviation of the errors is 0.05. We can see that both the mean and the standard deviation of the localisation error are less than 5% of r. For comparison, Figure 9 shows the localisation error without adding virtual nodes on the same original networks when there is 5% multiplicative ranging errors. A summary of the localisation errors in experiments of

Table 1 Localisation errors after adding virtual nodes when no noise is present. (m, s) are shown in each entry where m is mean and s is standard deviation

Random uniform ($r = 1.5$, $\bar{d} = 3.18$)	*Random uniform* ($r = 1.3$, $\bar{d} = 3.71$)	*Random C-shape* ($r = 1.3$, $\bar{d} = 4.44$)	*Random C-shape* ($r = 1.5$, $\bar{d} = 3.65$)
7.64e-7, 3.81e-7	1.86e-7, 3.39e-7	2.77e-7, 1.77e-7	3.66e-3, 2.34e-3

Figure 8 The localisation of four types of networks shown in Figure 7 when 5% ranging error and odometry error are considered. From left to right the four cases are regular uniform, random uniform, random C-shape, and regular C-shape. The circles are added for marking the estimated positions of nodes. The dotted line segments (most of them are too short to be clearly visible in this figure), indicate the localisation error. The nodes in these figures represent their locations after movement (see online version for colours)



Figures 8 and 9 is shown in Table 2. As we can see, there is significantly more error in the localisation if no virtual nodes are added.

One concern in the proposed approach is the noise introduced by the virtual nodes. The distances between virtual nodes can not be directly measured and the error introduced by the distances between virtual nodes will have a negative effect on localisation. To evaluate the effect of this noise and the noise in ranging measurement, we introduced various levels of noise on four networks of certain degree. In this experiment, all nodes whose inter-distance is less than r are connected. For each virtual node and each moving node, we add multiplicative noise to both the range measurement and the odometry according to the noise model described at the beginning of this section. In this experiment, the deviation of the noise varies from 0 to 0.15. The experiment is tried on four types of networks of average degrees less than 4. The mean and deviation of the error are shown in Figure 10. The r in the random uniform, regular uniform, regular in C-shape, and random in C-shape cases is 1.5, 1.4, 1.5 and 1.3 respectively. We can see that the localisation error increases as the ranging and odometry errors increase. We can also see that if the error is under 5% of r, the mean localisation error is smaller than 5% of r, which indicates that the proposed approach is quite reliable with respect to the errors in ranging and odometry. From these experiments, we can also see that the error introduced in the movement of nodes has significant effects on the localisation accuracy, which further justifies the necessity of the unique realisability test before each node's movement.

Figure 9 The localisation error without adding virtual nodes on the four networks shown in Figure 7. The red line segments, the known position to the estimated position, indicate the localisation errors. The nodes in these figures represent their locations after movement (see online version for colours)

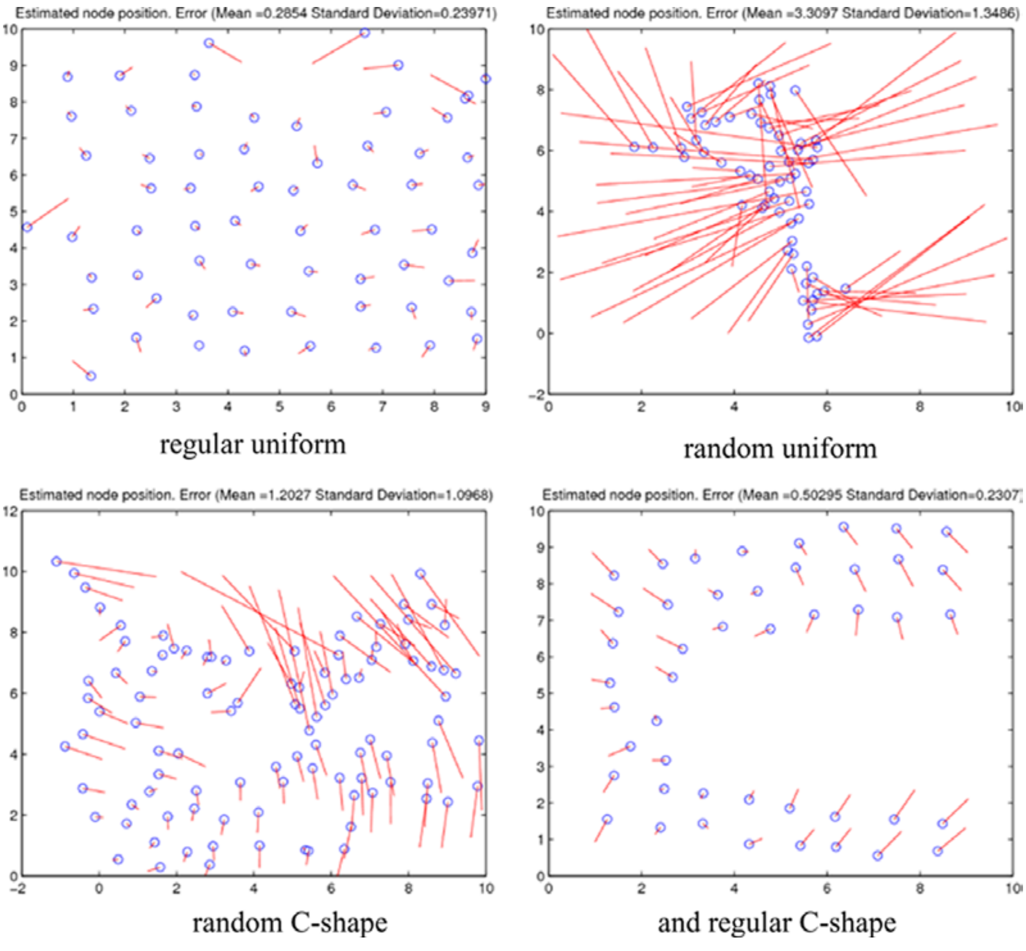


Table 2 Error comparison between with virtual nodes and without virtual nodes. $\bar{d}$ is the average degree of the network, (m, s) are shown in each entry where m is mean and s is standard deviation

	Random uniform ($r = 1.5$, $\bar{d} = 3.18$)	*Random uniform* ($r = 1.3$, $\bar{d} = 3.71$)	*Random C-shape* ($r = 1.3$, $\bar{d} = 4.44$)	*Random C-shape* ($r = 1.5$, $\bar{d} = 3.65$)
with virtual nodes	0.089, 0.051	0.038, 0.024	0.062, 0.032	0.046, 0.024
without virtual nodes	3.309, 1.348	0.285, 0.239	1.202, 1.096	0.502, 0.230

Figure 10 The effects of the error on the localisation accuracy in four types of networks. The average degrees of the networks are below 4

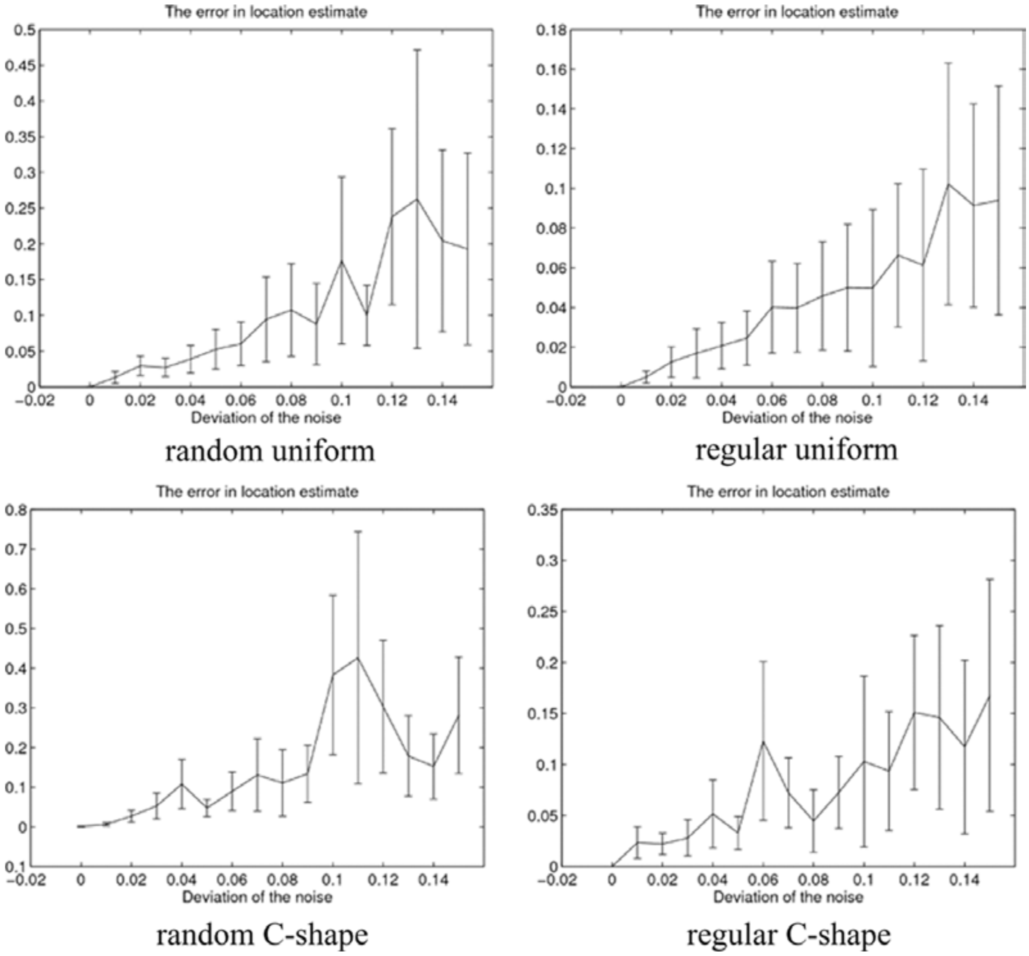


Figure 11 shows the result of an empirical analysis of the relationship between the number of moving nodes and the average degree of nodes in a network. In this experiment, the number of moving nodes in four types of networks of varying r is studied. With the changing of r, networks of different average degrees can be created. The number of moving node on each level of r for the four networks is counted. As we can see from this figure, when the average degree is around 4, more then 80% of the nodes in the networks have to move. The number of moving nodes decreases quickly with the increasing average degree. When the average degree increases to 6.5, the number of moving nodes decreases to around 50%. When the average degree increases to 9, the number of moving nodes is less than 30% of the total nodes in the network. Therefore, the proposed approach will save lots of time and energy in localising high-degree networks compared with the approach proposed in our previous work (Wu et al., 2006b).

We also conducted some evaluation on the number of moving nodes on already localisable networks. The motivation is to see how many nodes will be unnecessary moved if no global localisability is tested before applying the proposed method. We created a set of uniquely

Figure 11 The fraction of moving nodes with respect to the network degree. Cases 1, 2, 3 and 4 are four network topologies: random uniform, regular uniform, regular C-shape, and random C-shape respectively. As we can see when the network degree is 4, more than 80% of nodes has to move. When the network degree increases to 9, only less than 30% of nodes have to move

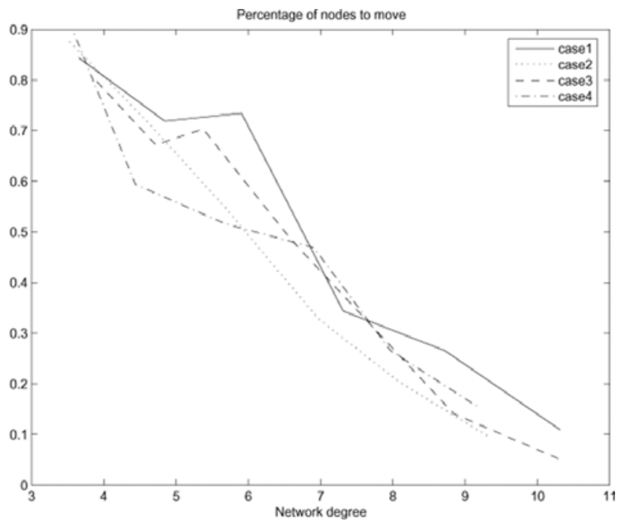


Figure 12 Networks that are uniquely realisable (see online version for colours)

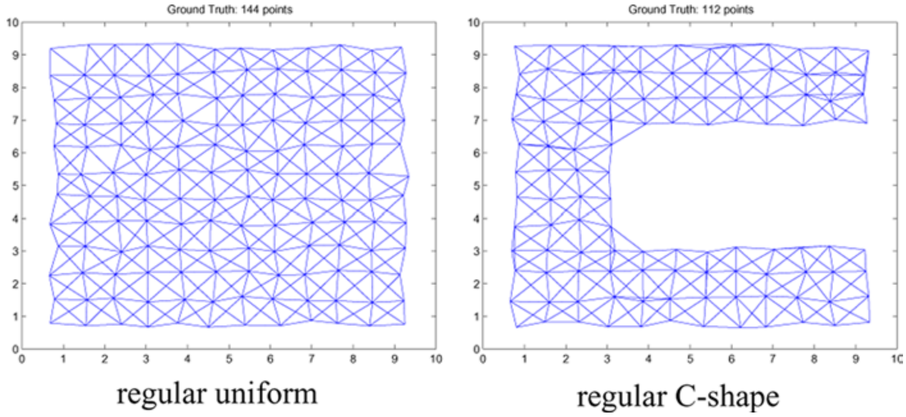
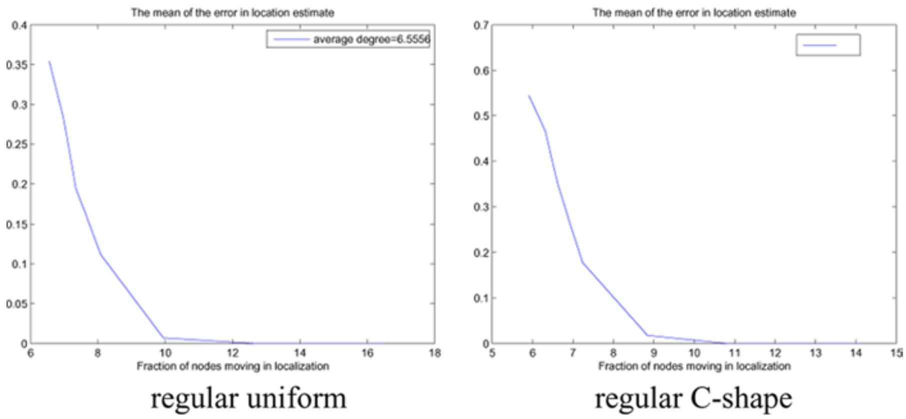


Figure 13 The fraction of moving nodes with respect to the network degree for networks that are already uniquely realisable (see online version for colours)



realisable networks. For simplicity reason, we only created networks for two topologies: regular uniform and regular C-shape. A small noise is added to the position of the nodes so the networks are not perfect grid. Figure 12 shows two network examples. The sensing range of the nodes are increased from 1.2 to 2.0 with an interval equal to 0.1 so that networks with different degrees can be generated. Figure 13 shows the change of number of moving nodes with respect to the network degree for regular uniform networks and regular C-shape networks. As we can see that with the degree increases, the number of moving nodes drops quickly, which is what we expected. We have also examined the allocation of the moving nodes and found that most of the moving nodes are located near the periphery of the networks.

7 Conclusion

In this paper, we introduce a rigidity-guided localisation approach for mobile robotic networks. In this approach, each node tests the unique realisability of the local neighbourhood before the movement. If the local neighbourhood is not uniquely realisable, it starts moving in a circular way around its original position and the radius of the circle is equal to the maximum distance within which the range measure is accurate.

The moving node adds virtual nodes when it first sees a neighbour and when a neighbour first disappears from its sight. We prove that, the local neighbourhood will become a uniquely realisable patch after adding the virtual nodes. The proposed approach has been evaluated on low-degree networks and on four-types of topologically different networks. The results have shown the effectiveness of the proposed approach.

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References

Akyildiz, I., Su, W., Sankarasubramaniam, Y. and Cayirci, E. (2002) 'A survey on sensor networks', *IEEE Commun. Mag.*, Vol. 40, pp.102–114.

Atallah, M.J. (1998) *Algorithms and Theory of Computation Handbook*, 1st ed., CRC Press, September 30.

Biswas, P. and Ye, Y. (2004) 'Semidefinite programming for ad hoc wireless sensor network localisation', *ACM/IEEE Conference Information Processing in Sensor Networks*, Berkeley, CA.

Borg, I. and Groenen, P. (1997) *Modern Multidimensional Scaling, Theory and Applications*, Springer-Verlag, New York.

Bulusu, N., Heidemann, J. and Estrin, D. (2000) *GPS-less Low Cost Outdoor Localisation for Very Small Devices*, Technical Report 00-729, Computer Science Department, University of Southern California, Los Angles, CA.

Cheng, K-Y., Lui, K-S. and Tam, V. (2006) *Hybrid Approach for Localisation in Anisotropic Sensor Networks*, Melbourne, Australia, Vol. 1, pp.344–348.

Cortes, J., Martinez, S., Karatas, T. and Bullo, F. (2004) 'Coverage control for mobile sensing networks', *IEEE Transactions on Robotics and Automation*, Vol. 20, No. 2, pp.243–255.

Costa, J., Patwari, N. and Hero, A. (2006) 'Distributed weighted-multidimensional scaling for node localization in sensor networks', *ACM Transactions on Sensor Networks* Vol. 2, No. 1, pp.39–64.

Doherty, L., Pister, K. and El Ghaoui, L. (2001) 'Convex position estimation in wireless sensor networks', *Proceedings of INFOCOM 2001, Twentieth Annual Joint Conference of the IEEE Computer and Communications Societies*, Anchorage, AK, Vol. 3, pp.1655–1663.

Eren, T., Goldenberg, D., Whitley, W., Yang, Y., Morse, A., Anderson, B. and Belheumer, P. (2004) 'Rigidity, computation, and randomisation of network localisation', *Proceedings of INFOCOM 2004, 23rd Annual Joint Conference of the IEEE Computer and Communications Societies*, Hong Kong.

Girod, L. and Estrin, D. (2001) 'Robust range estimation using acoustic and multimodal sensing', *Proceedings IROS 01, IEEE/RSJ International Conference on Intelligent Robots and Systems*, Maui, HI, Vol. 3, pp.1312–1320.

Goldenberg, D., Krishnamurthy, A., Maness, W., Yang, R., Young, A. and Savvides, A. (2005) 'Network localisation in partially localisable networks', *Proceedings of INFOCOM 2005, 24th Annual Joint Conference of the IEEE Computer and Communications Societies*, Miami, FL, Vol. 1, pp.313–326.

Hu, L. and Evans, D. (2004) 'Localisation for mobile sensor networks', *Proceedings of MobiCom'04*, Philadelphia, PA, pp.45–57.

Jackson, B. and Jordn, T. (2005) 'Connected rigidity matroids and unique realisations of graphs', *Journal of Combinatorial Theory Series B Archive*, Vol. 94, No. 1, pp.1–29.

Jacobs, D.J. and Hendrickson, B. (1997) 'An algorithm for two-dimensional rigidity percolation: the pebble game', *Journal of Computation Physics*, Vol. 137, pp.346–365.

Ji, X. and Zha, H. (2004) 'Sensor positioning in wireless ad-hoc sensor networks using multidimentional scaling', *Proceedings of INFOCOM 2004, 23rd Annual Joint Conference of the IEEE Computer and Communications Societies*, Hong Kong.

Kolodziej, K.W. and Hjelm, J. (2006) *Local Positioning Systems: LBS Applications and Services*. CRC Press, Taylor & Francis Group.

Laman, G. (1970) 'On graphs and rigidity of plane skeletal structures', *Journal of Engineering Mathematics*, Vol. 4, No. 4, October, pp.331–340.

Lim, H. and Hou, J.C. (2005) 'Localisation for anisotropic sensor networks', *Proceedings of INFOCOM 2005, 24th Annual Joint Conference of the IEEE Computer and Communications Societies*, Miami, FL, USA, Vol. 1, pp.138–149.

Lin, C-Y., Peng, W-C. and Tseng, Y-C. (2006) 'Efficient in-network moving object tracking in wireless sensor networks', *IEEE Transactions on Mobile Computing*, Vol. 5, No. 8, pp.1044–1056.

Liu, J., Chu, M. and Reich, J.E. (2007) 'Multitarget tracking in distributed sensor networks', *IEEE Signal Processing Magazine*, Vol. 24, No. 3, pp.36–46.

Mainwaring, A., Polastre, J., Szewczyh, R., Culler, D. and Anderson, J. (2002) 'Wireless sensor networks for habitat monitoring', *Proceedings of Wireless Sensor Network and Applications*, Atlanta, Georgia, USA.

Nguyen, X., Jordan, M. and Sinopli, B. (2005) 'A kernel-based learning approach to ad hoc sensor network localisation', *ACM Transactions on Sensor Networks*, Vol. 1, No. 1, pp.134–152.

Niculescu, D. and Nath, B. (2001) *Ad Hoc Positioning System (APS)*, San Antonio, TX, Vol. 5, pp.2926–2931.

Pathirana, P.N., Bulusu, N., Savkin, A. and Jha, S. (2005) 'Node localisation using mobile robots in delay-tolerant sensor networks', *IEEE Transactions on Mobile Computing*, Vol. 4, No. 4, pp.285–296.

Patwari, N., Ash, J.N., Kyperountas, S., Hero III, A.O., Moses, R.L. and Correal, N.S. (2005) 'Locating the nodes: cooperative localisation in wireless sensor networks', *IEEE Signal Processing Magazine*, Vol. 22, No. 4, pp.54–69.

Pfeifer, T. and Elias, D. (2003) 'Commercial hybrid ir/rf local positioning system', *Proceedings of Kommunikation in Verteilten Systemen (KiVS)*, Leipzig, Germany.

Priyantha, N.B., Balakrishnan, H., Demaine, E.D. and Teller, S. (2005) 'Mobile-assisted localisation in wireless sensor networks', *Proceedings of INFOCOM 2005, 24th Annual Joint Conference of the IEEE Computer and Communications Societies*, Miami, FL, Vol. 1, pp.172–183.

RevKoren, Y., Gotsman, C. and Ben-Chen, M. (2005) *Patchwork: Efficient Localisation for Sensor Networks by Distributed Global Optimisation Technical Report 35*, Technical Report, AT&T Research Lab., NJ, USA.

Savarese, C., Rabaey, J. and Beutel, J. (2001) 'Locationing in distributed ad hoc wireless sensor networks', *Proc. 2001 Int'l Conf. Acoustics, Speech, and Signal Processing (ICASSP 2001)*, Vol. 4, pp.2037–2040.

Shang, Y. and Ruml, W. (2004) 'Improved mds-based localisation', *Proceedings of INFOCOM 2004, 23rd Annual Joint Conference of the IEEE Computer and Communications Societies*, Hong Kong, pp.2640–2651.

Simic, S.N. and Sastry, S. (2003) 'Distributed environmental monitoring using random sensor networks', *Proceedings of the 2nd International Workshop on Information Processing in Sensor Networks*, Palo Alto, CA, pp.582–592.

Singer, A. (2008) 'A remark on global positioning from local distances', *Proceedings of the National Academy of Sciences*, Vol. 105, No. 28, pp.9507–9511.

Spletzer, J.R. (2003) *Sensor Fusion Techniques for Cooperative Localisation in Robot Teams*, PhD Thesis, University of Pennsylvania, Philadelphia, PA, USA.

Taylor, C., Rahimi, A. and Bachrach, J. (2006) 'Simulatenous localisation, calibration and tracking in an ad hoc sensor network', *5th International Conference on Information Processing in Sensor Networks (IPSN06)*, Nashville, Tennessee, pp.27–33.

Tilak, S., Kolar, V., Abu-Ghazaleh, N.B. and Kang, K.D. (2005) 'Dynamic localisation control for mobile sensor network', *Performance, Computing, and Communications Conference, IPCCC 2005, 24th IEEE International*, April pp.587–592.

Tsai, H-W., Chu, C-P. and Chen, T-S. (2007) 'Mobile object tracking in wireless sensor networks', *Computer Communications*, Vol. 30, No. 8, pp.1811–1825.

Tseng, Y-C., Kuo, S-P., Lee, H-W. and Huang, C-F. (2004) 'Location tracking in a wireless sensor network by mobile agents and its data fusion strategies', *Computer Journal*, Vol. 47, No. 4, pp.448–460.

Vercauteren, T. and Wang, X. (2005) 'Decentralised sigma-point information filters for target tracking in collaborative sensor networks', *IEEE Transactions on Signal Processing*, Vol. 53, No. 8, pp.2997–3009.

Vercauteren, T., Guo, D. and Wang, X. (2005) 'Joint multiple target tracking and classification in collaborative sensor networks', *IEEE Journal on Selected Areas in Communications*, Vol. 23, No. 4, pp.714–723.

Wang, C., Ding, Y. and Xiao, L. (2006) 'Virtual ruler: mobile beacon based distance measurements for indoor sensor localisation', *2006 IEEE International Conference in Mobile Ad hoc and Sensor Systems (MASS), 2006*, pp.326–335.

Wang, X. and Wang, S. (2007) 'Collaborative signal processing for target tracking in distributed wireless sensor networks', *Journal of Parallel and Distributed Computing*, Vol. 67, No. 5, pp.501–515.

Wu, C., Sheng, W. and Zhang, Y. (2006a) 'Mobile self-localization using multi-dimensional scaling in robotic sensor networks', *The International Journal of Intelligent Control and Systems*, Vol. 11, No. 3, pp.163–175.

Wu, C., Sheng, W. and zhan Song, W. (2006b) 'A dynamic MDS-based localisation algorithm for mobile sensor networks', *Proceedings of IEEE International Conference on Robotics and Biomimetics*, Kunming, China, pp.496–501.

Wu, C., Sheng, W. and Zhang, Y. (2007) 'Mobile sensor networks self localisation based on multi-dimensional scaling', *Proceedings of 2007 IEEE International Conference on Robotics and Automation ICRA'07*, Roma, Italy, pp.4038–4043.

Zhang, F., Pereira, G.S. and Kumar, V. (2005) 'Cooperative localisation and tracking in distributed robot-sensor networks', *Tsinghua Science and Technology*, Vol. 10, No. 1, pp.91–101.

Zhang, L., Liu, L., Gotsman, C. and Gortler, S.J. (2009) *An As-Rigid-as-possible Approach to Sensor Network Localization*, Technical Report, Harvard Computer Science Technical Report: TR-01-09.

Zhao, F. and Guibas, L. (2004) *Wireless Sensor Networks: An Information Processing Approach*. Elsevier and Morgan Kaufmann Publishers, ISBN: 978-1558609143.

Zou, Y. and Chakrabarty, K. (2007) 'Distributed mobility management for target tracking in mobile sensor networks', *IEEE Transactions on Mobile Computing*, Vol. 6, No. 8, pp.872–887.