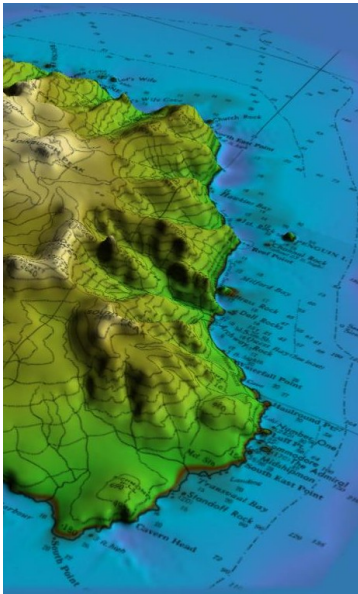




# A Geometrical Perspective on Localization



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- > A second look at the localization problem
- > Cramer-Rao bound revisited
- > Noise-resilience metric
- > Conclusions

> Problem:

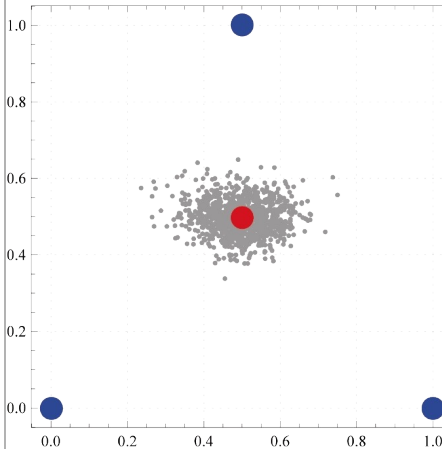
- > Given a set of nodes with known positions (anchors)
- > Given (noisy) distance measurements to a node
- > Find the position of the node

> "Usual" solution

- > (Multi)-iteration to compute the actual position
- > Residuals for on-line estimate of precision
- > GDOP factors to assess effects of geometrical setup

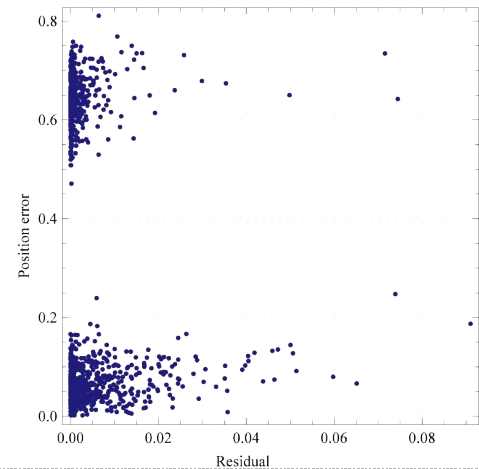
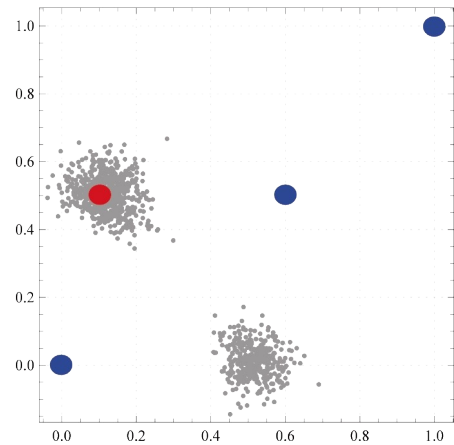
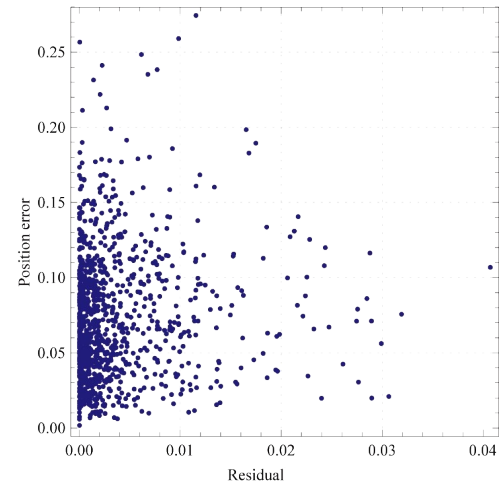
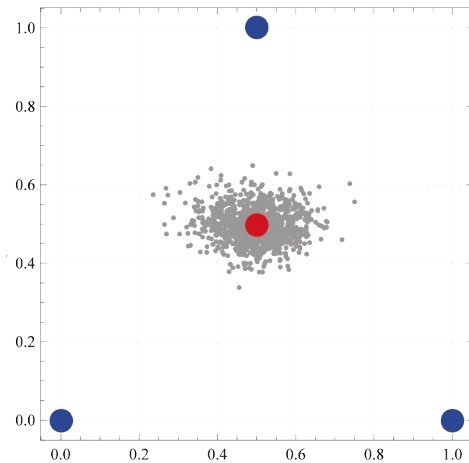
> Assumptions

- > Small residual values lead to smaller position errors
- > Anchor collinearity leads to situations with no solutions

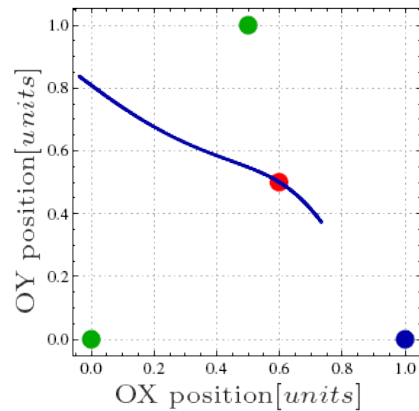


$$\tilde{z} = \arg_{\tilde{z}} \min \sum_{i=1}^n \omega_i (d_i - \tilde{d}_i)^2$$

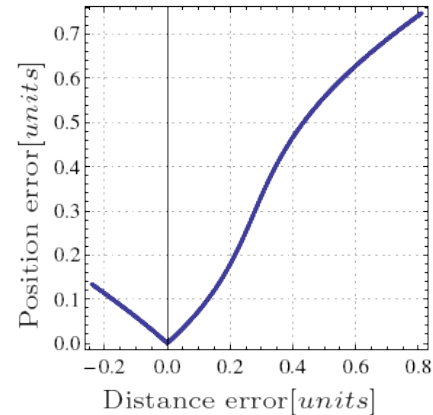
# Residual values vs. precision



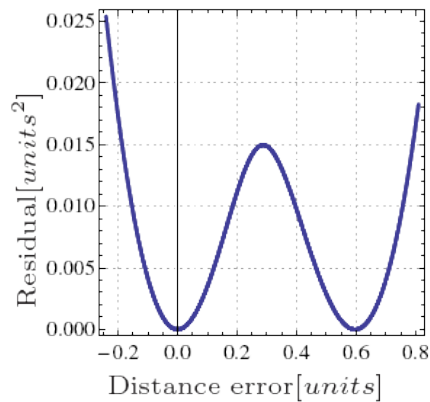
# Residual value behavior



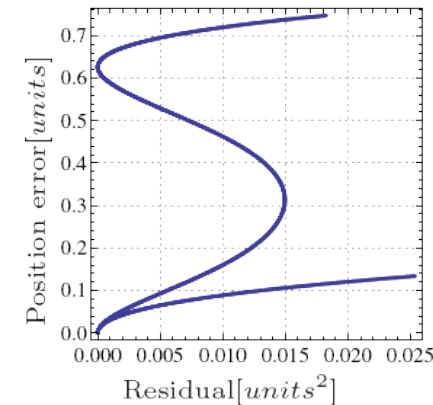
(a) Computed position trajectory



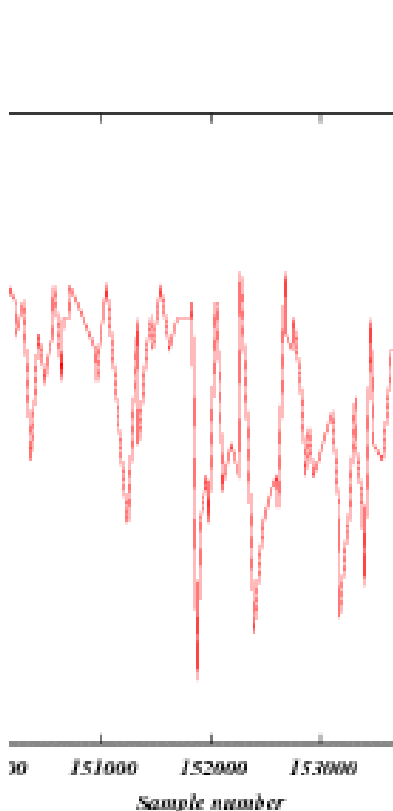
(b) Position errors vs. distance error



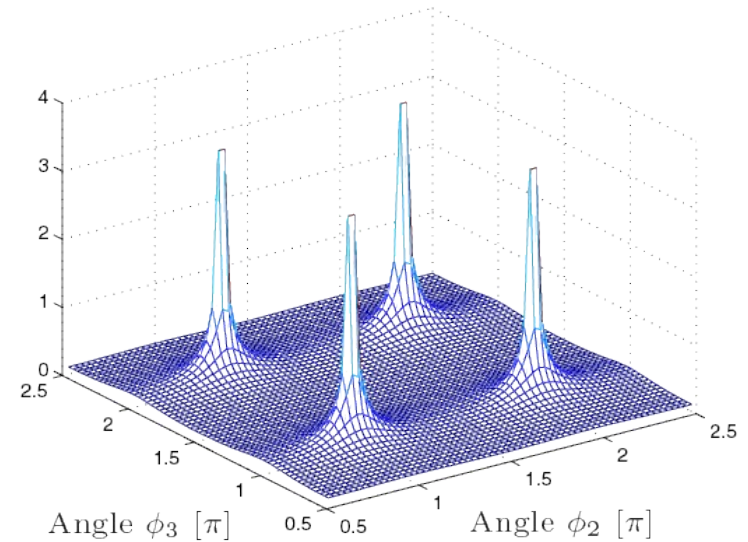
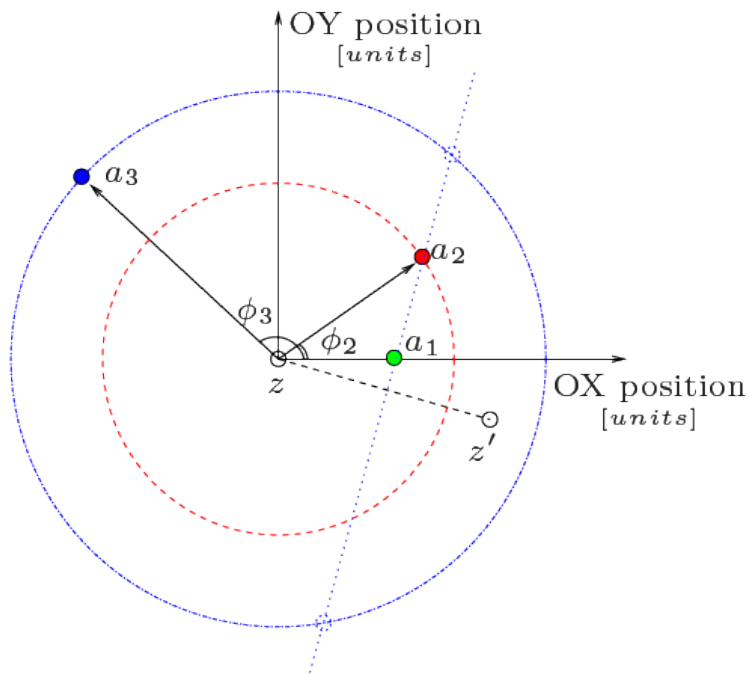
(c) Residual vs. distance error



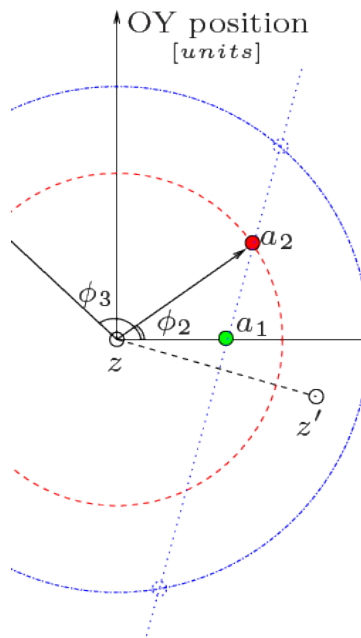
(d) Position error vs. residual



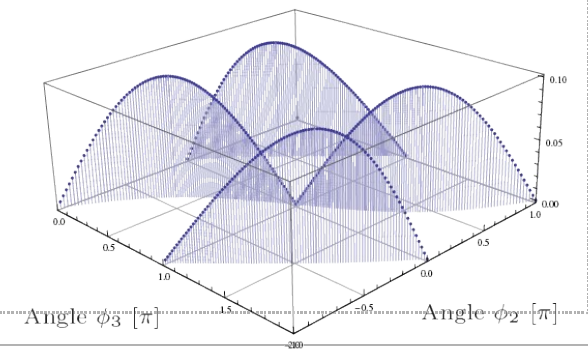
- > Statistic boundary determining the minimum achievable variance of any unbiased estimator
  - > Useful metric used with success in many fields
  - > Applied to localization systems for both single-hop and multi-hop localization
  - > From its formulation, in certain cases (GPS) the geometrical setup influence can be taken apart as the GDOP coefficient

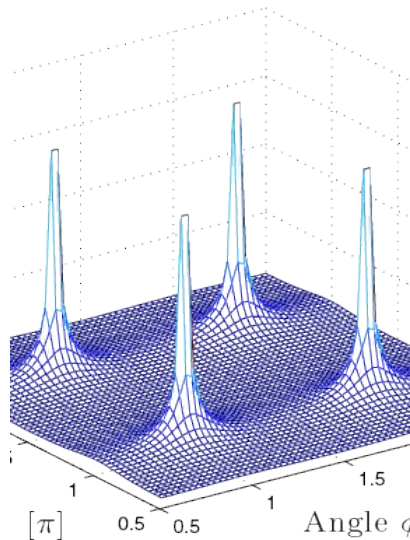


Cramer Rao bound

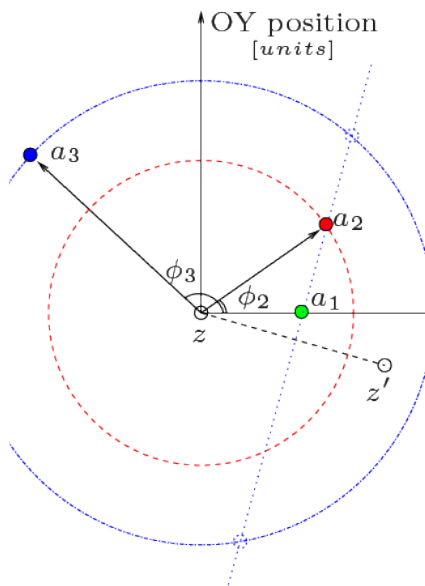


- > Define the distance  $zz'$  as *flipping uncertainty*
- > The experiment shows that in case of collinearity, uncertainty goes to infinity
  - > A subtlety: collinearity of anchors AND the node
- > Flipping uncertainty is 0 when node is collinear with anchors
  - > In case of collinearity of anchors, the node being on the same line is the only situation when position can be computed
  - > This contradicts insight given by CRB
- > Not captured by CRB:
  - > There are many more situations in which anchors are collinear, not only 4



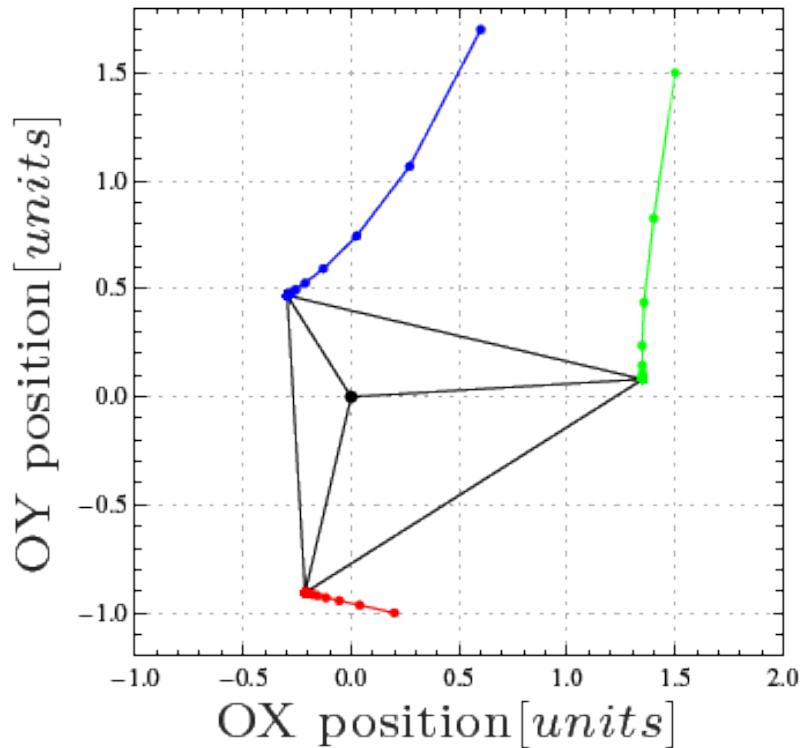


- > GDOP derived from CRB under simplifications:
  - > Mean values of distance measurements are in the same order of magnitude
  - > Variances on distance measurements are similar
- > Neither simplification holds in sensor networks
- > CRB does not behave correctly in sensor networks, thus GDOP should also not be relied on

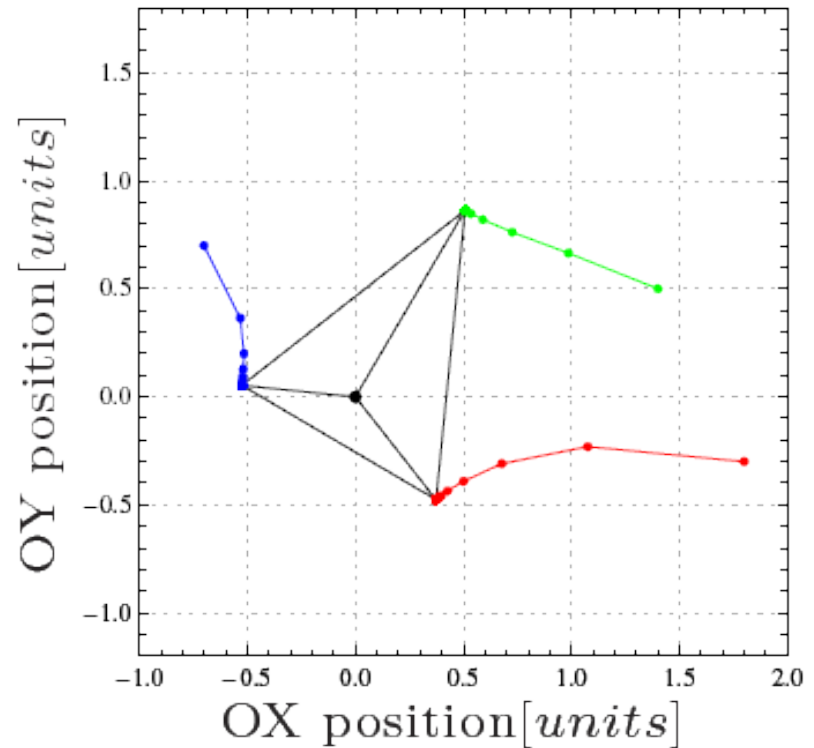


- > Define *noise resilience*:
  - > Consider three anchors and one node
  - > Consider the line defined by two anchors
  - > Consider node ( $z$ ) and its reflection in the line ( $z'$ )
  - > Consider distances  $z$ -third anchor and  $z'$ -third anchor ( $d$  and  $d'$ )
  - > Noise resilience =  $\text{abs}(d-d')$
- > Noise resilience shows how *stable* a topology is
  - > The smaller the value, the smaller the amount of noise one can add to obtain flipping situations

# Noise resilience & anchors



(a) Initial positions – example 1

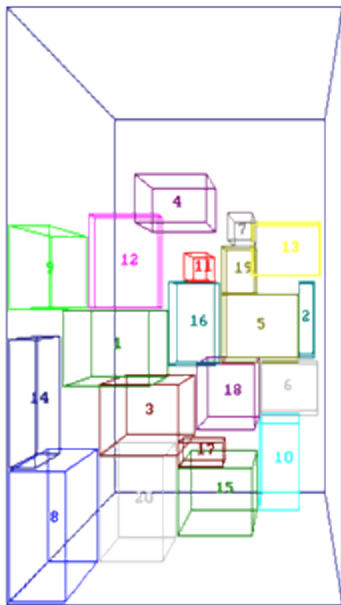


(b) Initial positions – example 2

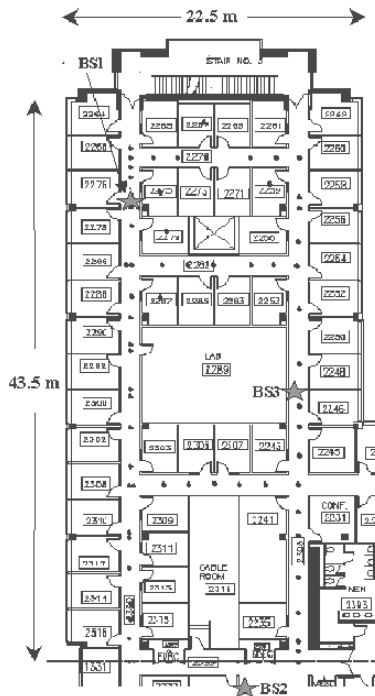
Iterative anchor placement by minimizing noise resilience at each step



- > Define distance functions
  - > Given anchor positions ( $a_i$ ) and node position ( $z$ )
  - > Define distances ( $d_i$ )
  - >  $d_i$  are uniquely defined, procedure leads from positions to distances (call it  $f$ )
- > Define  $f^{-1}$ 
  - > Given distances ( $d_i$ )
  - > Given anchor positions ( $a_i$ )
  - > Compute position of the node ( $z$ )
- > Main issue:
  - >  $f^{-1}$  is undefined for collinear anchors
    - > Function associates only one value to some input
  - >  $f^{-1}$  needs to be specified in all situations



- > Basic concept: geometrical setup (positions of anchor and node)
- > Theoretical step: define  $f^{-1}$ ; determine geometrical boundaries for what is achievable
- > Algorithm: from these boundaries, derive metric to be minimized
- > Post processing: assuming parameters of noise known, refine boundaries using statistical techniques as CRB



- > Conclusions
  - > CRB and GDOP are to be applied with caution in geometrical configurations provided by sensor networks
  - > Residual value used as online estimator should be also treated carefully
  - > It all depends on geometry...
- > Future work
  - > Implement the new conceptual flow
  - > Explore the definitions of  $f^{-1}$  and possible new algorithms (error metrics?)

# Thank you!



> Questions?

