

Mathematics of Sensor Network Localization

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Joint work with Biswas, So, Toh, Boyd, Wang, Zheng, Jin, Saunders,

...

Outline

- Sensor Network Localization and Graph Realization
- Mathematical Formulation and Analysis
- Computational Demonstrations
- Where we are now

Graph Realization

Given a graph $G = (V, E)$ and sets of non-negative **weights**, say $\{d_{ij} : (i, j) \in E\}$ and $\{\theta_{ilj} : (i, l, j) \in \Theta\}$, the goal is to compute a **realization** of G in the **Euclidean space** \mathbf{R}^d for a **given low dimension** d , i.e. to place the vertices of G in \mathbf{R}^d such that

- the **Euclidean distance** between every pair of adjacent vertices (i, j) equals (or bounded) by the prescribed weight $d_{ij} \in E$;
- the **angle** between edges (i, l) and (j, l) equals (or bounded) by the prescribed weight $\theta_{ilj} \in \Theta$.

The data can be time-series, and it can be treated as an additional dimension with additional distance constraints.

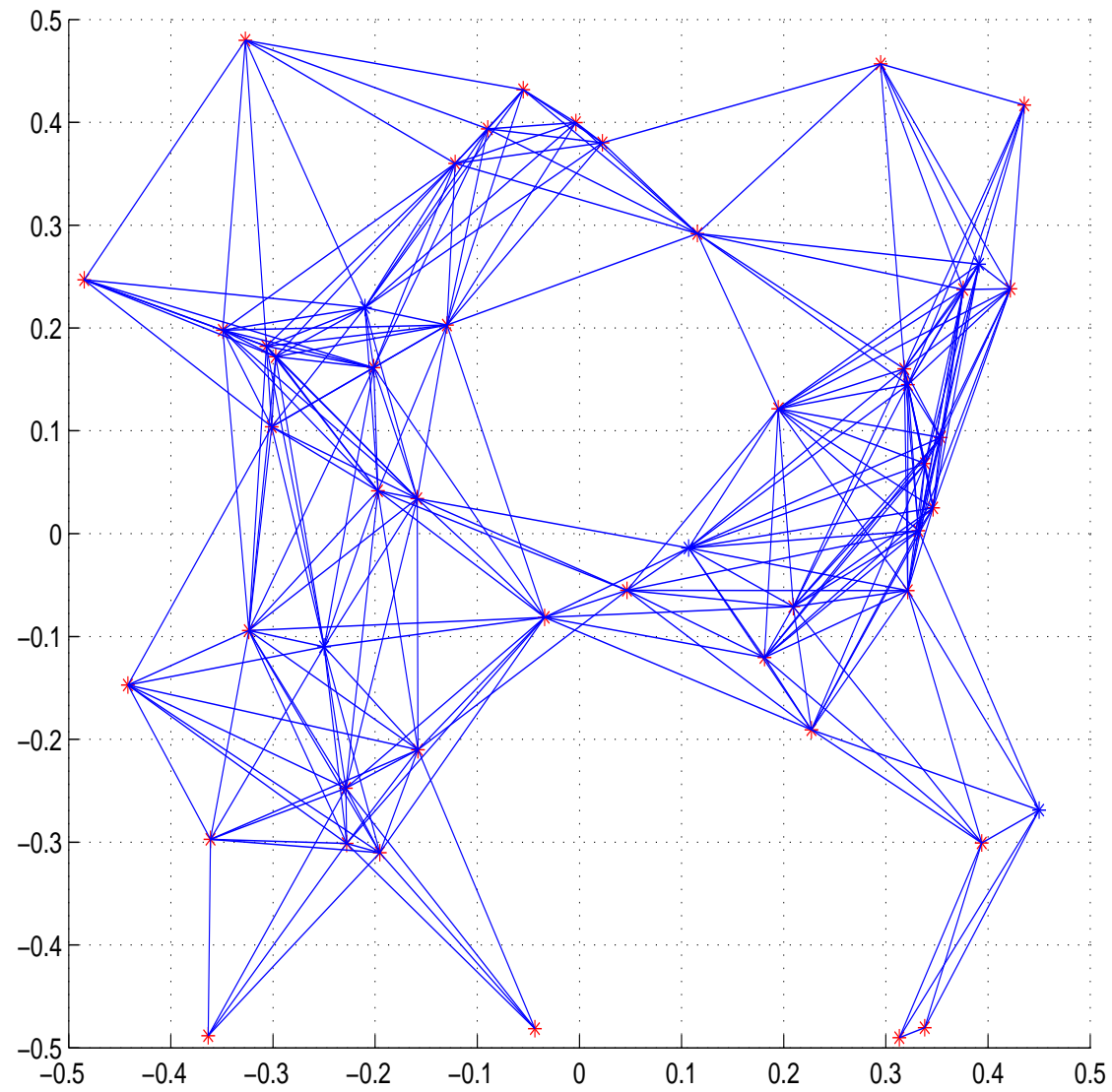


Figure 1: 50-node 2-D **Sensor Localization**

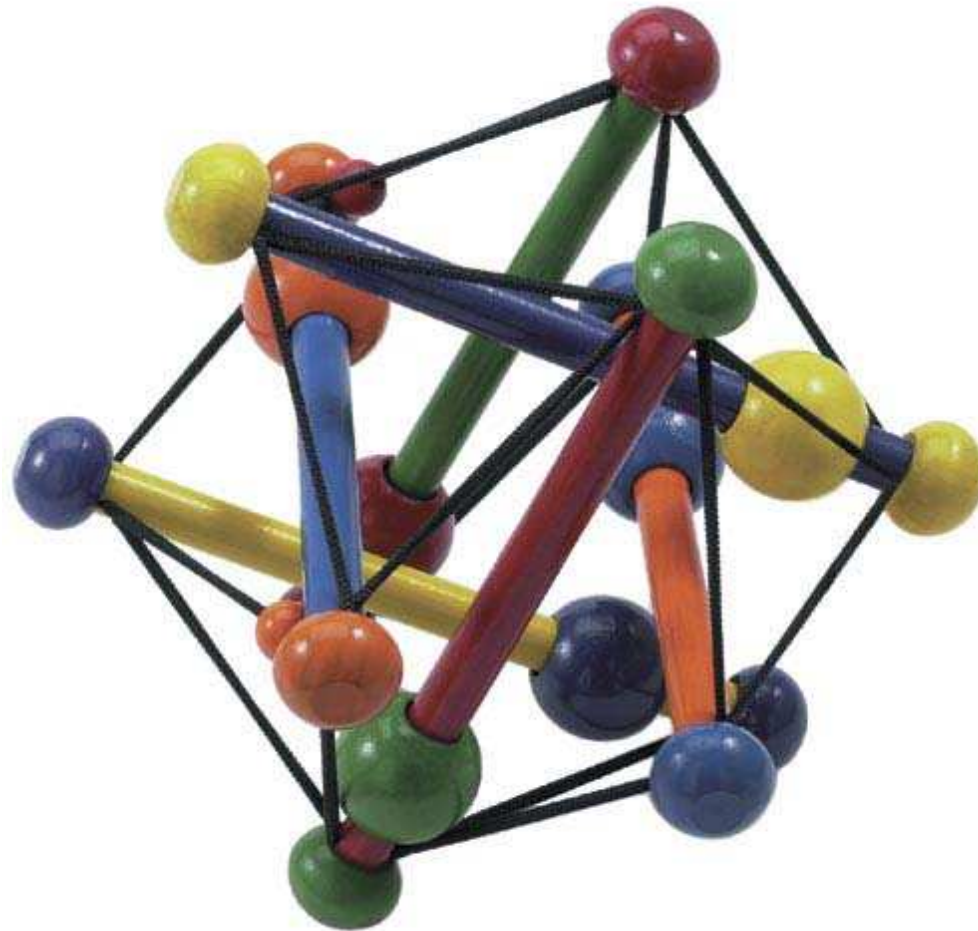


Figure 2: A 3-D **Tensegrity Graph** Realization; picture provided by Anstreicher

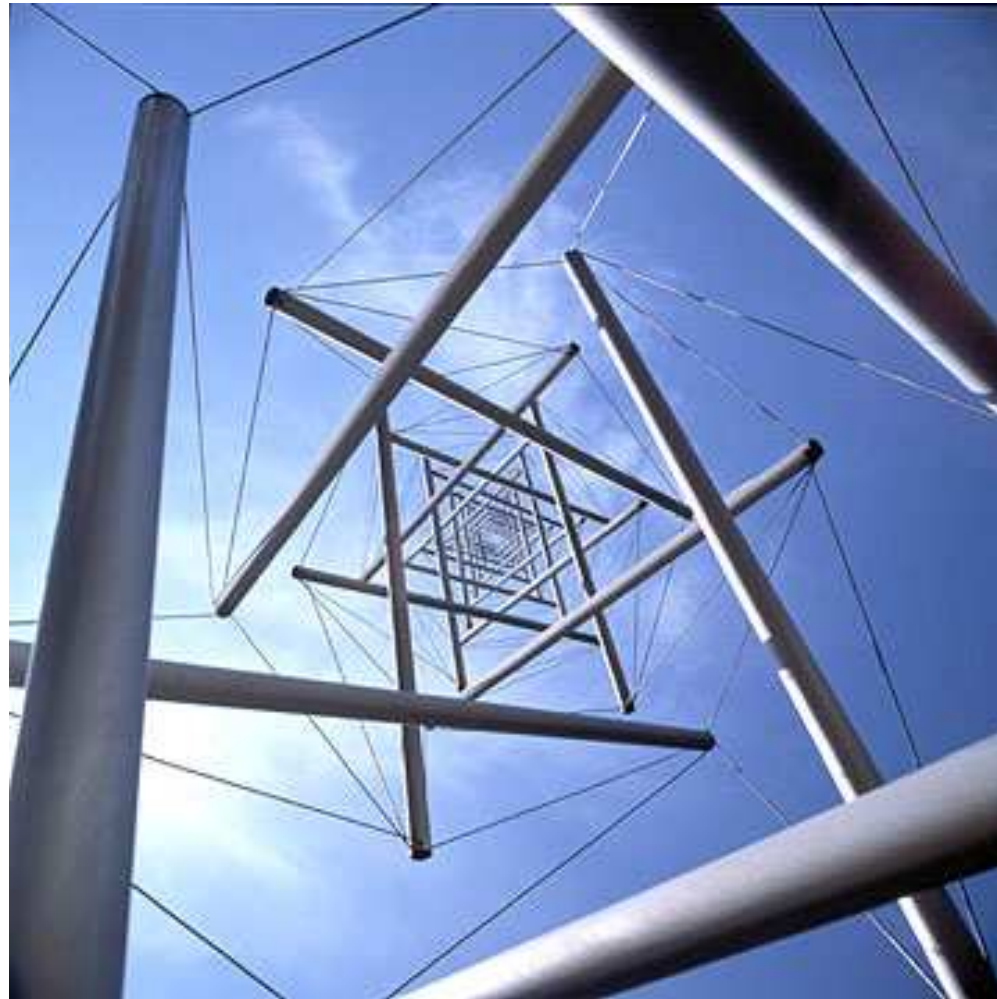


Figure 3: **Tensegrity Graph**: A Needle Tower; picture provided by Anstreicher

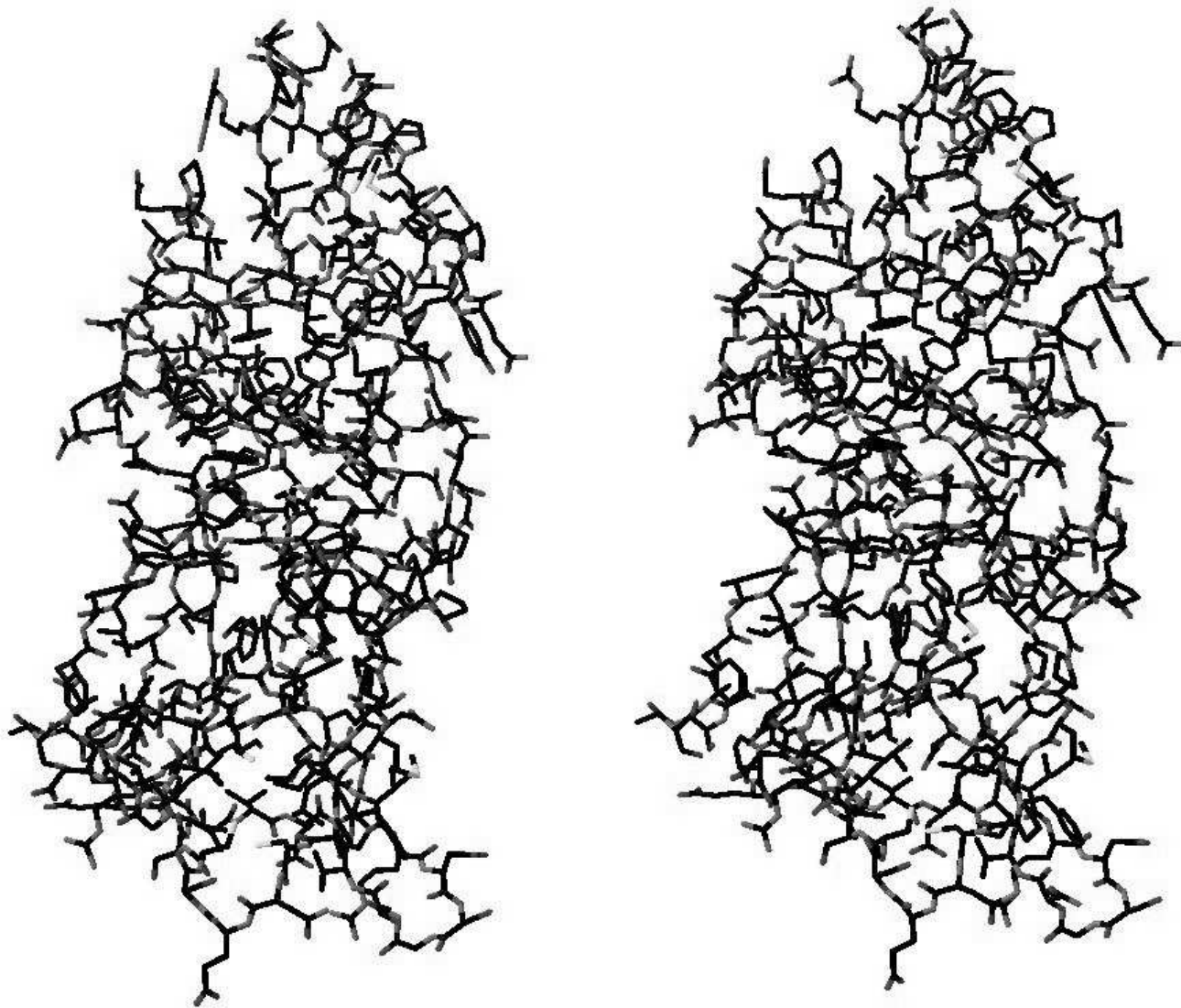


Figure 4: **Molecular Conformation:** 1F39(1534 atoms) with 85% of distances below 6\AA and 10% noise on upper and lower bounds

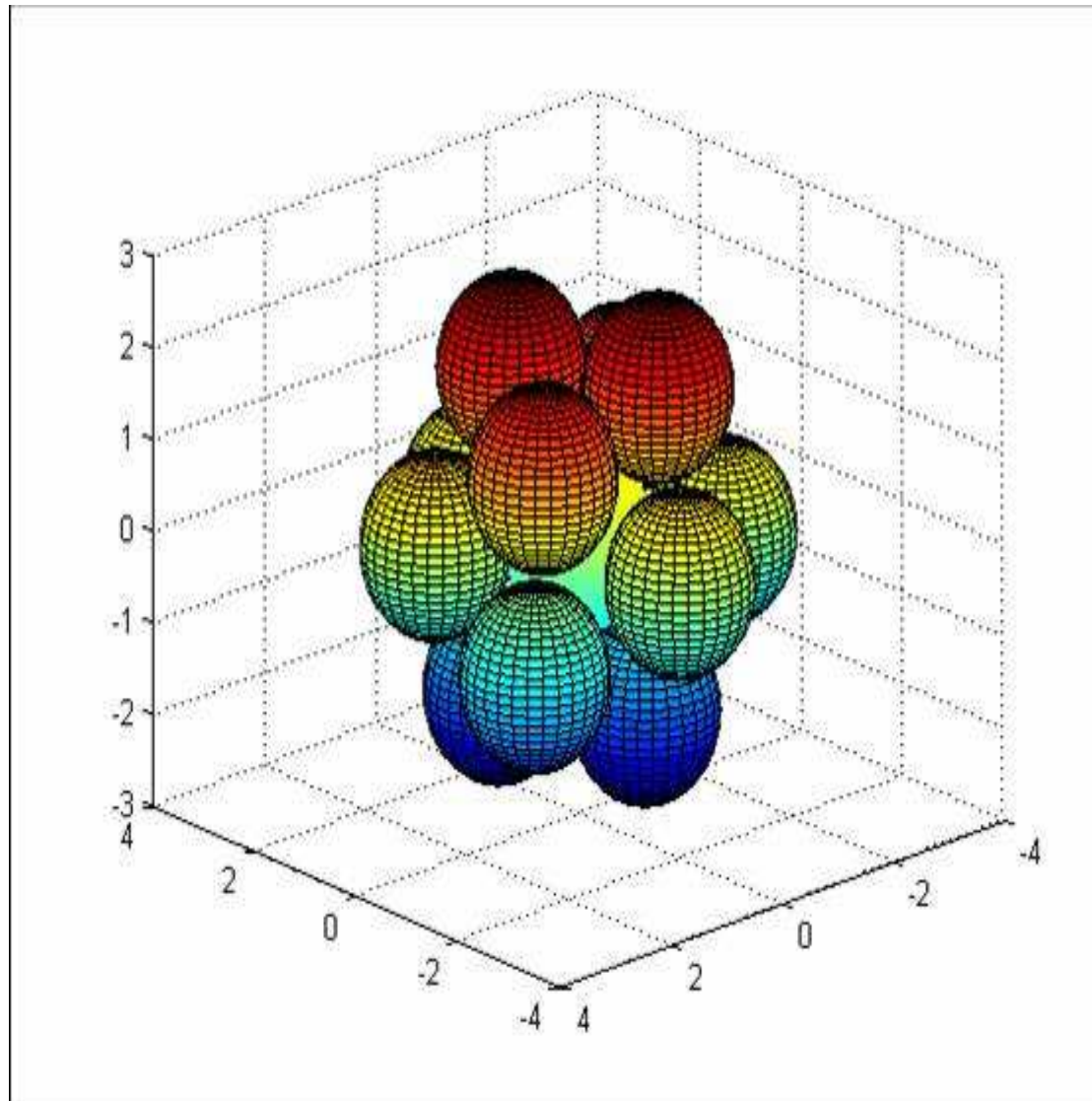


Figure 5: The 3-D Kissing Problem

Related Mathematical Work

- **Multidimensional scaling**: Schoenberg and Young/Householder (1932) studied the case where all pair-wise distances are given;
- **Metric embedding**: Johnson and Lindenstrauss (1984) and Bourgain (1985); **Graph Realization**: Connelly and Sloughter (2004);
- Barvinok, Pataki, Alfakih/Wolkowicz and Laurent (1995-) used semidefinite programming (SDP) model to solve the problem when the **dimension** of localization is not restricted;
- if the realization is required to be in \mathbf{R}^d for some **fixed** d , then the problem becomes **NP-complete** (e.g., Saxe 1979, Aspnes, Goldenberg, and Yang 2004);

SDP and SOCP for Localization

More recently, using SDP and second-order cone programming SOCP (Doherty et al. 01, Biswas and Y 04, So and Y 05, Tseng 05, Biswas, Toh, and Y 06, Jin et al. 06, Wolkowicz et al. 06, Wang et al. 06, Kojima et al. 08, etc.) to

- identify **families of graph instances** that admit polynomial time algorithms for computing a localization in the **required dimension**,
- develop **theories** for sensor network realization, and
- produce effective methods for solving **large-scale and practically noisy** localization problems

(IPSN'04, SODA'05, SODA'06, ACM, IEEE, Math. Programming, SIAM J. ...).

Mathematical Formulation: Quadratic Problems

Given $a_k \in \mathbf{R}^d$, $d_{ij} \in N_x$, and $\hat{d}_{kj} \in N_a$, find $x_i \in \mathbf{R}^d$ such that

$$\|x_i - x_j\|^2 \quad (\leq) = (\geq) \quad d_{ij}^2, \quad \forall (i, j) \in N_x, \quad i < j,$$

$$\|a_k - x_j\|^2 \quad (\leq) = (\geq) \quad \hat{d}_{kj}^2, \quad \forall (k, j) \in N_a,$$

(ij) ((kj)) connects points x_i and x_j (a_k and x_j) with an edge whose Euclidean length is d_{ij} (\hat{d}_{kj}).

Does the system have a localization or realization of all x_j 's? Is the localization **unique**? Is there a **certification** for the solution to make it **reliable or trustworthy**? Is the system **partially** localizable with certification?

“Rigid Graph” can be Realized in Polynomial Time

Theorem 1. *(So and Y 2005)*

1. *The sensor network that is d -uniquely-localizable can be localized in polynomial time by solving an SDP relaxation model;*
2. *the SDP model computes a solution that localize all possibly localizable sensor points;*
3. *the SDP model generates confidence information to tell which points are localized accurately and which are not.*

Figure 6: One sensor-Two anchors: Not localizable

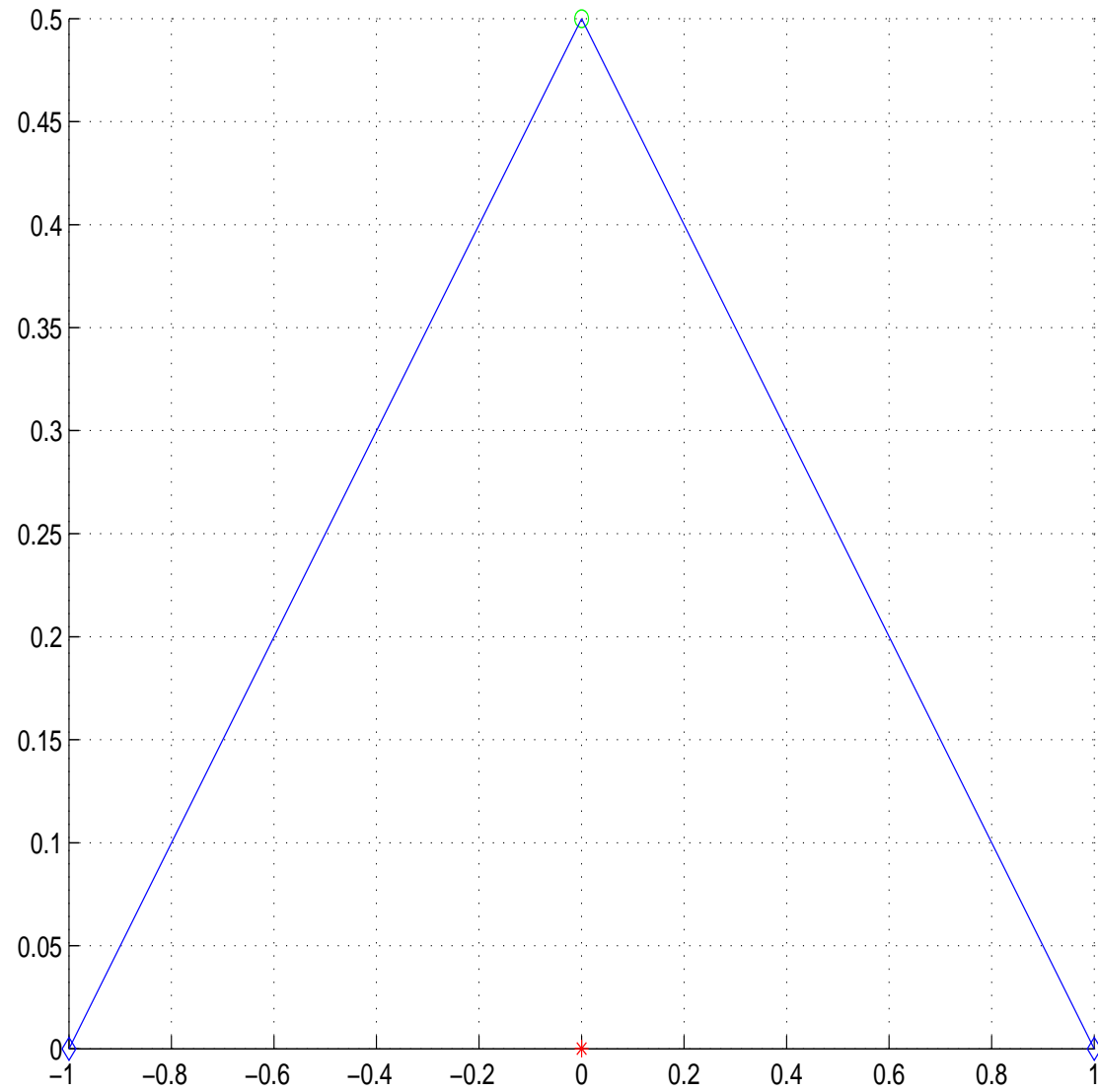


Figure 7: Two sensor-Three anchors: Uniquely Localizable

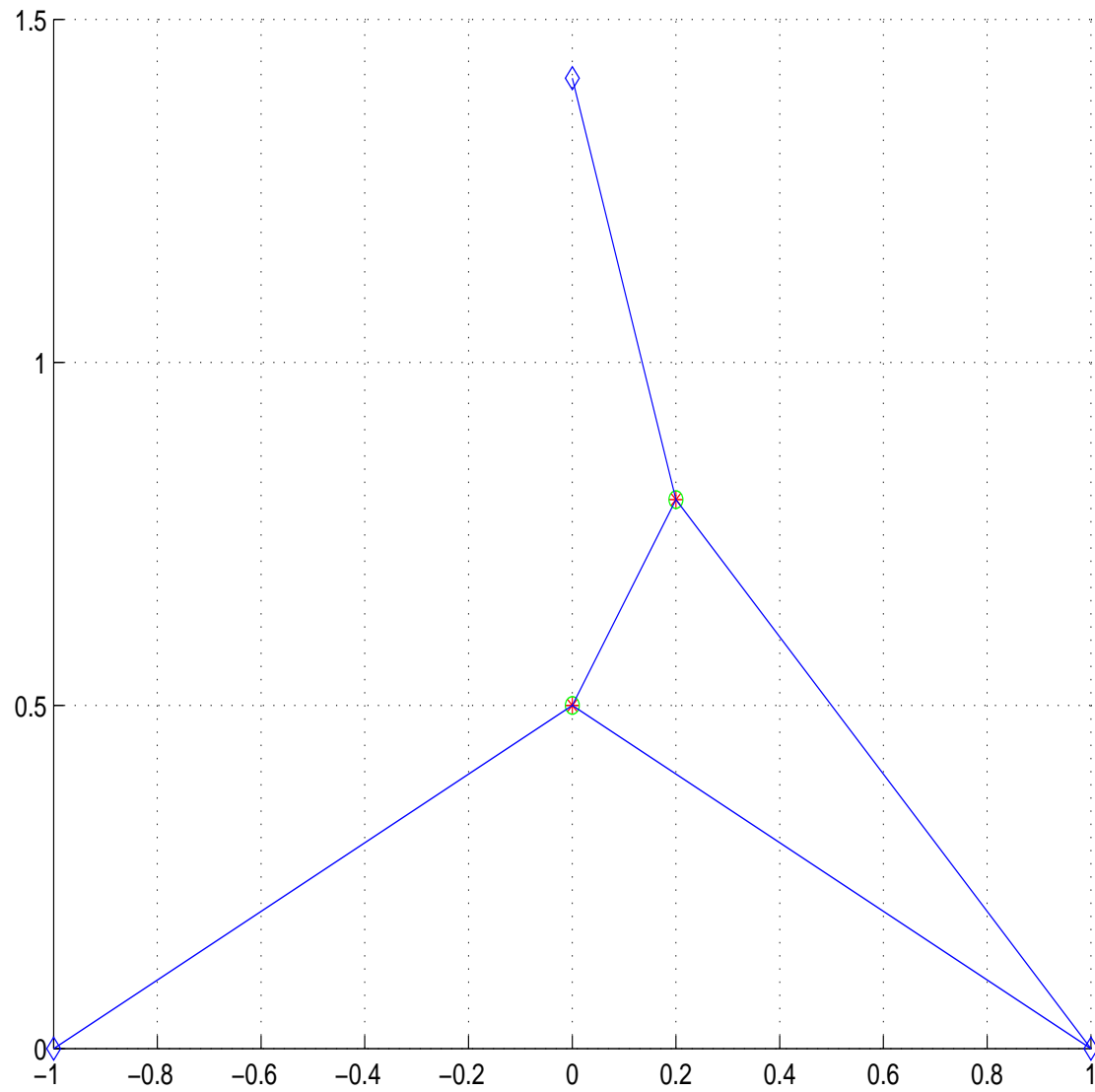
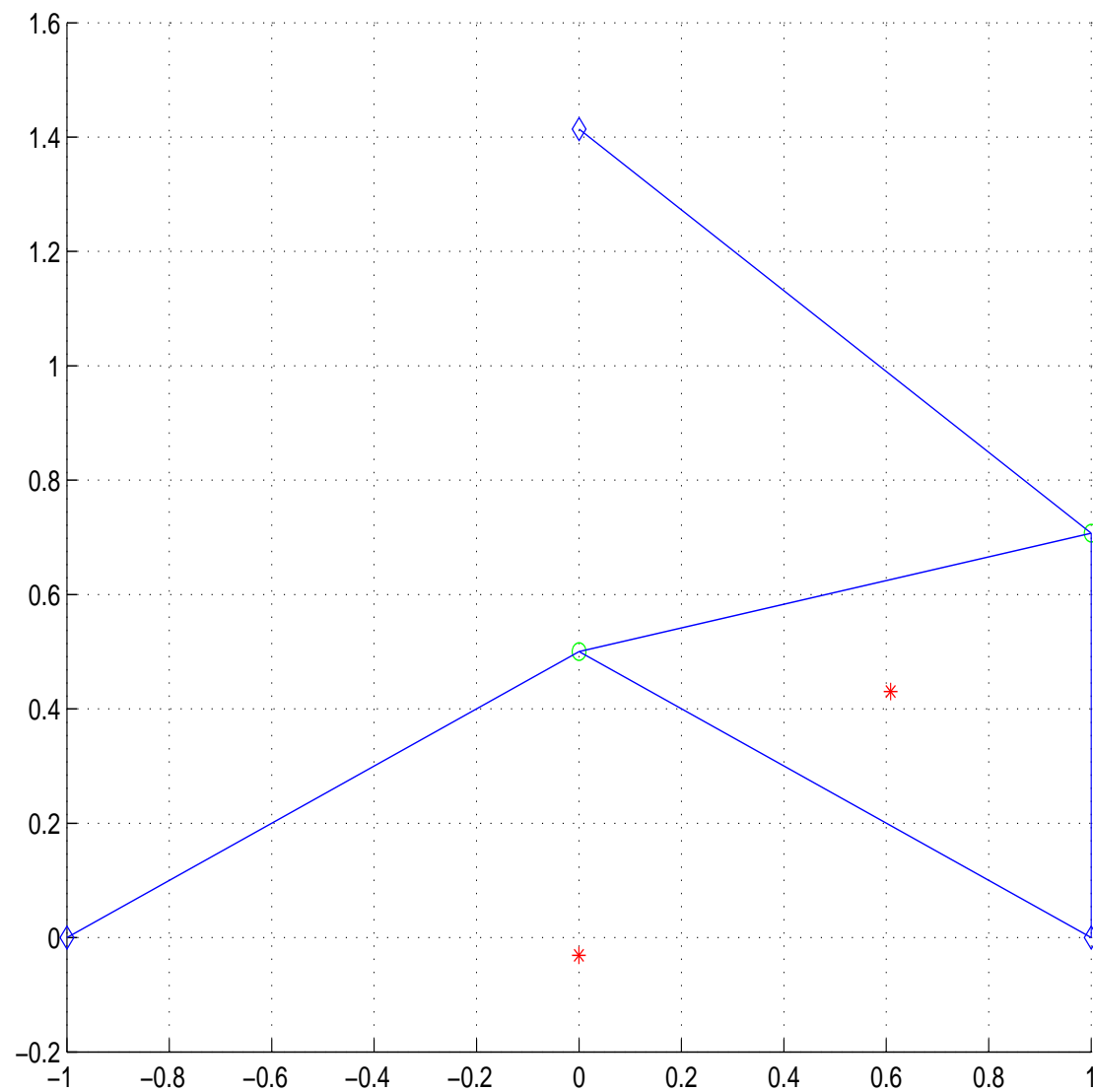


Figure 8: Two sensor-Three anchors: Not localizable



Matlab SDP Computation

SDP solvers used were SeDuMi (Sturm, 2001) and DSDP2.0 (Benson et al. 1998).

In the computational experiments:

$$d_{ij} = \text{trued}_{ij} \cdot (1 + \text{randn}(1) \cdot nf)$$

We construct SDP models to minimize **max-likelihood** errors when distance measures are noisy (Biswas 2007).

Where We are Now

- Solving sensor network with **hundreds** points and limited edge information in seconds.
- Producing accurate position estimations in a **noisy environment**.
- **Distributed and decentralized** methods localize large-scale networks in seconds.
- **Tracking** fast moving sensors at high speed.

Reference Sites

- <http://www.stanford.edu/~yye/>
- <http://orion.uwaterloo.ca/~hwolkowi/>
- <http://www.math.nus.edu.sg/~mattohkc>
- <http://www.math.washington.edu/~tseng/>
- <http://www.is.titech.ac.jp/~kojima/>

Talks in Session

Mathematical optimization and/or maximal likelihood model that match sensor locations with observed measurable data in different forms.

- “Cooperative Localization using Angle of Arrival Measurements in Non-line-of-sight Environments” investigates localization of a transmitting node using **angle of arrival** (AoA) measurements made at a geographically dispersed network of cooperating receivers with known locations.
- “Localization of Mobile Users Using Trajectory Matching” develops an algorithm enabling localization of moving wireless devices in an indoor setting by estimating a mobile nodes geographical **trajectory** given its time series signal strength measurements.
- “Sensor Network Localization, Euclidean Distance Matrix Completions, and Graph Realization” studies Semidefinite Programming model to localize sensors simply based on a subset of **pair-wise distance data**.